Modelling of extremes
Application on electricity day-ahead spot prices time series

Igor Paholok

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Modelling of extremes
Application on electricity day-ahead spot prices time series

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Abstract

Mean reversion model, jump diffusion mean reversion model and extreme value theory concept were applied in this article in order to calculate Value at Risk and Conditional Value at Risk figures. Calibration of models and their predictive power were backtested on a sample of very volatile electricity day-ahead spot prices time series. The paper’s main objective is to provide a comparison of selected models.

Keywords:
Mean reversion model, Jump diffusion mean reversion model, Extreme value theory, Value at Risk, Conditional Value at Risk

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1. Introduction

The recent financial crisis (2008-2009) disclosed a serious shortcoming in financial modelling issues and risk measure concepts. Models calibrated on historical time series failed in the prediction of recent extremes. Furthermore, the Value at Risk (VaR) concept excludes the extreme values with regard to selected confidence level. Therefore, the Conditional Value at Risk (CVaR) concept has became more and more relevant as it measure the average value of the tail over the selected confidence level. VaR and CVaR concepts are applied in this essay as well. The value of VaR and CVaR will be calculated using three methods (mean reverting process, jump diffusion and mean reverting process, extreme values theory) over period of 2008-2009 (series from previous two years 2006-2007 were used for starting models calibration) and back-tested on daily electricity spot prices of three selected time series. Spot contracts with electricity have been chosen for their higher volatility in comparison with other underlying instruments and for extreme price jumps occurrence. Applied methods might be employed on any similarly behaved time series as well.

The paper’s main objective is to provide a comparison of three models that might be used in the process of VaR and CVaR calculation. Each of the selected models might be applied, but final results would probably vary. This variation should demonstrate how crucial the process of model selection might be.

Essay contains basic theory explanation including brief literature reviews, analysis of data characteristics, descriptions of model calibration, back-testing results, model suitability assessment and resume.

1.1 Literature review

The mean reverting model and the jump diffusion mean reverting model are relatively well known concepts. Among others, I can mention references as Culot, Goffin, Lawfords [4] and Meyer-Brandis, Tankov [11] and Cartea, Marcelo [3] with close focus on seasonal decomposition before modelling. Model calibration technics are described in Garcia Franco [8] and Dixit, Pindyck [7] for mean reverting model, in Lyzanets, Senchyna [10] for jump diffusion mean reverting model and in Baran [2], Lyzanets, Senchyna [10] for extreme value theory. Mean reverting model and jump diffusion mean reverting model are routinely used for electricity spot prices stochastic modelling in contrast with EVT where the usage is not very common within this specific area.
2. Models and concepts theory

2.1 Mean reverting process

The mean reverting process (e.g. Ornstein – Uhlenbeck process) is frequently used in order to provide commodity prices modelling especially for its economic logic. The process is defined by the following stochastic differential equation.

\[ dx(t) = \eta(x - x(t))dt + \sigma \, dB(t); \]  

(2.1)

Where \( x(t) = \ln(P_t), \) \( \eta \) represents the speed of mean reversion, \( x \) is the long-run equilibrium level, \( \sigma \) is volatility and \( B(t) \) is regular Brownian notion.

Pattern of model’s drift is given by the difference between long-run equilibrium and current price level, and by the mean reversion speed. If the current price level is higher than the long-run equilibrium the drift is negative and vice versa. The difference is multiplied by the reversion speed. Process also consists of stochastic part given by Brownian notion and volatility.

Model parameters might be estimated by several ways. Maximum likelihood estimation (presented for example by García Franco [7]) runs optimization process (e.g. Quasi-Newton) using \( x_{t-1} \) conditional density function of \( x_t, \) for \( t-1 < t \), which is given by

\[ f(x_t; \bar{x}; \eta; \sigma) = (2\pi) \frac{1}{2} \left( \frac{\sigma^2}{2\eta} \right) \exp \left[ \frac{(x_t - \bar{x} - (x_{t-1} - \bar{x})e^{-\eta(t-(t-1)))})^2}{2\sigma^2(1-e^{-2\eta(t-(t-1)))}} \right]; \]  

(2.2)

The alternative approach presented by Dixit and Pindyck [6] solves optimization problem considering \( dx(t) \) as a first-order autoregressive (AR1) process. For the limiting case (\( dt \) tends to zero) we can write the process as

\[ x_t - x_{t-1} = \bar{x}(1 - e^{-\eta \Delta t}) + (e^{-\eta \Delta t} - 1)x_{t-1} + \varepsilon_t; \]  

(2.3)

Where \( \varepsilon_t \) is normally distributed with mean zero and standard deviation \( \sigma \) and

\[ \sigma^2 = \frac{[1-\exp(-2\eta)]\sigma^2}{2\eta}; \]  

(2.4)

We can obtain model parameters by following regression

\[ x_t - x_{t-1} = a + bx_{t-1} + \varepsilon_t; \]  

(2.5)

Where

\[ \bar{x} = \frac{-a}{b}; \]  

(2.6)
\[ \eta = -\ln(1 + b); \quad (2.7) \]
\[ \sigma = \sigma + \frac{2\ln(1 + b)}{(1 + b)^2 - 1}; \quad (2.8) \]

2.2 Jump diffusion mean reverting process

Jump diffusion mean reverting process is suitable for modelling of occasional price jumps with subsequent mean reverting. This is the typical pattern of electricity day-ahead spot prices. The process is given by following equation

\[ dx(t) = \mu dt + \sigma dt B(t) + Jdq; \quad (2.9) \]

Where \( \mu \) is equal with \( \eta (x - x(t)) \) from equation (3.9), \( \sigma \) is the volatility, \( J \) is the mean of jump size. \( dq \) presents the Poisson process, where \( dq = 1 \) with probability \( \lambda \) and \( dq=0 \) with probability \( 1- \lambda \). The jump size is assumed to be normally distributed with the volatility \( \sigma \) and mean \( J \).

Regarding Lazanets and Senchyna [9], the model calibration could be realized by maximum likelihood estimation using the following density function:

\[ f(x) = \sum_{n=0}^{\infty} \frac{e^{-\lambda} \lambda^n}{n! \sqrt{2\pi(\sigma^2 + \sigma_j^2)}} \exp \left( \frac{- (x - x - nJ)^2}{2(\sigma^2 + n\sigma_j^2)} \right); \quad (2.10) \]

Another, let’s say “analytical approach” is a method, when we separate extreme values which are used for jump parameters calibration (\( J, \sigma_j, \lambda \)). The residual time series is used for calibration of models drift part (\( \eta, \bar{x}, \sigma \)). The question about jumps separating method remains. On the other hand, pure statistical maximum likelihood estimation might leads to results which are difficult to explain.

2.3 Extreme value theory

Extreme value theory (EVT) is a more general concept in comparison with mean reverting and jump diffusion mean reverting model presented in previous sections. It focuses on extreme values modelling using data which is available about extreme events. Therefore, EVT might be used when the time series tails distribution does not fulfill the normal distribution characteristics (e.g. fat tails).

Extreme values are selected by an appropriate approach. For instance, peak-over-thresholds assumes that values above specific threshold do not contain any information about the frequency and magnitude of extreme values observation. Those extreme values are used in the process of specific distribution calibration regardless of under-threshold data.

The Generalized Pareto Distribution (GPD) describes the limit distribution of scaled excesses over the thresholds. The GPD for returns time series \( r \) is defined as:
\[ G_{\xi,\alpha,r}(x) = 1 - \left( 1 + \frac{r - \mu}{\beta} \right)^{-\xi}; \text{if } \xi \neq 0 \]  
\text{and} 
\[ G_{\xi,\alpha,r}(x) = 1 - \left( 1 + \exp \left( \frac{r - \mu}{\beta} \right) \right); \text{if } \xi = 0 \]  

Where \( \xi \) (the tail index) is the parameter of the shape and \( \beta \) is the parameter of the scale. \( \mu \) presents the threshold value.

Underlying distribution functions (for the large enough \( \mu \)) become

\[ F_x(r) \approx (1 - F_x(\mu)) \left[ 1 - \left( 1 + \frac{r - \mu}{\sigma} \right)^{-\xi} \right] + F_x(\mu) \]  
\text{and} 
\[ F_x(r) \approx \frac{N_{\mu}}{n} \left( 1 - \left( 1 + \frac{r - \mu}{\sigma} \right)^{-\xi} \right) + 1 - \frac{N_{\mu}}{n} \]  

Where \( n \) is the number of sample observations and \( N_{\mu} \) is the number of excesses over threshold parameter \( \mu \).

2.4 VaR and CVaR concepts

At a given confidence level \( \alpha \), the Value at Risk is defined as the worst possible realization \( r^* \) of asset returns \( r \) such the probability of exceeding this value is \( 1-\alpha \) on a given time horizon:

\[ 1 - \alpha = \int_{-\infty}^{x^*} f(r) \, dr = P(r \leq r^*) = p; \]  

Value at Risk ignores returns which exceed value on a given confidence level. This fact is considered as concept’s disadvantage, therefore the concept Conditional Value at Risk becomes more and more popular. Value of CVaR (also called as Expected shortfall) is defined as an expected value (mean) of extreme returns which exceed Value at Risk on a given confidence interval.

Applying extreme value theory, the VaR as \( 1-\alpha \) distribution of losses is calculated from distribution function as defined in equation 2.14:

\[ \text{VaR}_\alpha = \mu + \frac{\beta}{\xi} \left( \left( \frac{n}{N_{\mu}} \right)^{\frac{1}{\xi}} - 1 \right); \]
All variables were defined in previous chapter (2.3). Expected shortfall (CVaR) is defined as:

\[ CVaR = VaR_{\alpha} + \frac{\beta + \xi (VaR_{\alpha} - \mu)}{1 - \xi}; \]

(2.17)

### 2.5 Backtesting

We assess the model accuracy by backtesting procedure which appraises applied model according to number of VaR violations. VaR figures are compared to observed data and number of expected excesses is given by selected VaR confidence level. Confidence level for model acceptance is not defined as expected value but as a specific confidence interval derived from inverse Poisson distribution with specific confidence intervals \((\alpha_{1} = (1 - \alpha)/2 \text{ and } \alpha_{2} = 1 - \alpha_{1})\) according to Papež [13]. Described backtesting method is not capable to discover possible model’s risk overestimation. Therefore we could measure the overestimation as a separate figure:

\[
\begin{align*}
  r_{o,1} &= \sum_{i=1}^{n} \sqrt{(VaR_{\text{upr}}(t) - r(t))^2} \text{ for } r(t) > 0 \\
  r_{o,2} &= \sum_{i=1}^{n} \sqrt{(VaR_{\text{lowr}}(t) - r(t))^2} \text{ for } r(t) < 0 \\
  r_{o} &= r_{o,1} + r_{o,2}
\end{align*}
\]

(2.18)

Where \(r_{o}\) is measure of risk overestimation. \(r_{o,1}\) represents sum of residuals between \(VaR_{\text{upr}}\) (Value at Risk calculated as a upper quantile of \(\alpha\), applied for example for short positions) and realized positive returns. \(r_{o,2}\) represents sum of residuals between \(VaR_{\text{lowr}}\) (Value at Risk calculated as a lower quantile \(1-\alpha\)) and realized negative returns. \(r_{o}\) measures expansion between realized returns and both sides VaR (or CVaR) figure. The lower \(r_{o}\), the better is model fit, having number of VaR excesses constant.
3. Time series characteristics

We test presented models on the selected day ahead electricity spot time series. The selection has been made randomly, considering available time series with at least four years history (from 1.1.2006 until 31.12.2009). Some time series had to be excluded for theirs trading day calendar specifics. Especially peak load contracts are not traded during weekend on several markets and traded on the others. Potentiality of our framework is strictly limited by positive last price of used time series. Crossing from positive to negative price level (and vice versa) excludes logarithm (which we apply to logarithmic returns calculation) from real numbers set. Similarly, the daily change with zero value start leads to infinite results. Selected time series are:

- APX Power NL Day ahead 24-hour electricity time average spot price (APX BL)
- Powernext Power Exchange day ahead base-load electricity average (POWERNEXT BL)
- European Energy Exchange Peak Load Phelix Electricity spot price (PHELIX PL)

Electricity spot contracts time series are usually very volatile, with extreme jumps occurrence, seasonal patterns and distribution with non normal characteristics. These characteristics are derived from natures of underlying instrument as seasonal demand, limited storability or even weather dependence.

Prices development of analyzed time series, values of daily changes, histograms, autocorrelation functions and partial autocorrelation functions are shown in Appendix A. We can intuitively endorse our models selection as we see that prices tend to reverse to long term mean after occasion jump diffusion.

Following sub-chapters focus on normality tests and seasonal decomposition.

3.1. Normality tests

We used Lilliefors test of normality and we can refuse the null hypothesis of normality for all studied time series within p-value 0.01. Test results could be supported by skewness and kurtosis statistics:

Table 3.1: Time series skewness and kurtosis

<table>
<thead>
<tr>
<th></th>
<th>APX BL</th>
<th>POWERNEXT BL</th>
<th>PHELIX BL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skewness</td>
<td>0.5</td>
<td>0.94</td>
<td>0.86</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>6.5</td>
<td>12.37</td>
<td>5.02</td>
</tr>
</tbody>
</table>

Source: Author’s analysis
Positive skewness indicates that extreme returns/gains are higher than extreme losses. Fat tailed density is evident from zoom on distribution tails. Next figure is an illustrative example of APX BL data distribution compared to normal density distribution.

3.2. Seasonal decomposition

Autocorrelation and partial autocorrelation function (Appendix A) of all examined time series indicate strong seasonal pattern on seven days (or seven days multiply) lags. Therefore we suggest use following seasonal function:

$$x(t) - x(t-1) = \mu_1 + \mu_2 WD_{1,7} + \mu_3 WD_{2,7} + \mu_4 WD_{3,7} + \mu_5 WD_{4,7} + \mu_6 WD_{5,7} + \mu_7 WD_{6,7} + \mu_8 WD_{7,7} + \epsilon_t; \quad (3.1)$$
Where \( \mu_i \) are seasonal factors and \( WD_{it} \) is 7-by-7 matrix with diagonal 1 for \( i = 1,2,3...7 \) according t-weekday (1 refers to Monday...7 refers to Sunday). Seasonal factors are estimated using least square error, minimizing sum of \( e_t \). Seasonal factors for all reference series (period from 1.1.2006 until 31.12.2009) are:

Table 3.2: Seasonal factors

<table>
<thead>
<tr>
<th></th>
<th>( \mu_1 )</th>
<th>( \mu_2 )</th>
<th>( \mu_3 )</th>
<th>( \mu_4 )</th>
<th>( \mu_5 )</th>
<th>( \mu_6 )</th>
<th>( \mu_7 )</th>
<th>( \mu_8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>APX BL</td>
<td>-0.00079</td>
<td>0.103088</td>
<td>-0.01656</td>
<td>-0.01123</td>
<td>-0.04843</td>
<td>-0.18179</td>
<td>-0.22631</td>
<td>0.380452</td>
</tr>
<tr>
<td>POWERNEXT BL</td>
<td>-0.00063</td>
<td>0.068195</td>
<td>-0.01511</td>
<td>-0.00443</td>
<td>-0.05555</td>
<td>-0.18262</td>
<td>-0.24037</td>
<td>0.429267</td>
</tr>
<tr>
<td>PHELIX BL</td>
<td>-0.00015</td>
<td>0.048421</td>
<td>-0.00165</td>
<td>-0.03232</td>
<td>-0.08622</td>
<td>-0.25682</td>
<td>-0.26667</td>
<td>0.5950</td>
</tr>
</tbody>
</table>

Source: Author’s analysis

Residuals, unseasonal returns, are listed in Appendix B. Autocorrelation functions do not indicate as strong seasonal patterns as original returns series did. Illustrative comparison of APX returns sample and APX returns sample given by applied seasonal function is shown on following chart:

![Figure 3.2: Comparison of APX returns sample with APX seasonal returns sample](image)

Residuals are not normally distributed as well. After we segregate seasonal part of observed returns we can follow concept.
\begin{align}
    r(t) = r_{\text{seasonal}}(t) + r_{\text{unseasonal}}(t); \\
    \text{Where } r(t) = x(t) - x(t - 1). \text{ We model seasonal part } r_{\text{seasonal}}(t) \text{ using function (3.1) and } r_{\text{unseasonal}}(t) \text{ is modeled separately using mean reverting model, mean reverting jump diffusion model and extreme value theory.}
\end{align}
4. Models Application

4.1 Models Calibration

Four years history gives use 1460 observations of daily returns (unseasonal returns) for each time serie. First 730 observations are used for initial model’s calibration. Next calibrations are proceeded continually, with one day moving, 730 days history.

Mean reverting model parameters are estimated for AR(1) process (2.3) using equations (2.4 – 2.8). Series of estimated parameters for last two years (730 days) of each examined return series are listed in Appendix C.

Mean reverting jump diffusion process parameters are estimated using “analytical approach” which is explained in section 2.2. Every daily return over 99 % quantile, for 730 day history, is considered as a jump. Selected jumps are used for jumps mean \( J \) and jumps variance \( \sigma_J \) estimation. Residual time series are used for mean reverting parameters estimation. Final parameters are presented in Appendix D.

Extreme value theory parameters shape \( (\xi) \) and scale \( (\beta) \) were estimated using maximum likelihood estimation applying MATLAB function for EVT parameters estimation. Results of moving calibration process are attached in Appendix E.

Estimated parameters of all applied models are stable and remarkable change occurs mostly when a new extreme observation gets in or old extreme observation gets out of the moving 730 days history band.

4.2 VaR and CVaR calculation

VaR and CVaR are calculated for both sides, for losses (VaR\(_{\text{lower}}\)) as well as for gains (VaR\(_{\text{upper}}\)), providing that we do not know whether we keep long or short position. Figures are calculated for observed returns and consists of seasonal part, which is modeled as expected value of the function (3.1) and residual – unseasonal part, which is modeled as 0.99 quantile (upper quantile) and 0.01 quantile (lower quantile) using one of tree examined model and Monte Carlo simulation. Whole procedure can be formalized as

\[
\text{VaR}(t) = r_{\text{seasonal}}(t) + \text{VaR}_{\alpha/1-\alpha}^\text{mod el}(t); \tag{4.1}
\]

and

\[
\text{CVaR}(t) = r_{\text{seasonal}}(t) + \text{CVaR}_{\alpha/1-\alpha}^\text{mod el}(t); \tag{4.2}
\]

Mean reverting model, mean reverting jump diffusion model and extreme value theory are subsequently used as a model applied.
4.3 Results overview

Having 730 historical observations we proceed backtesting according section 2.5. With 99\% confidence level we accept model if recorded violations of \( \text{VaR}_{\text{lower}} \) figures stay within interval <1,15>. Same interval is applied for \( \text{VaR}_{\text{upper}} \) figures. Overview of VaR excesses is presented in table 4.3 a table of CVaR excesses is attached in Appendix I.

Table 4.3: Number of VaR excesses

<table>
<thead>
<tr>
<th>Model</th>
<th>( \text{VaR}_{\text{lower}} ) excesses</th>
<th>( \text{VaR}_{\text{upper}} ) excess</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABX BL mean reverting</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>POWERNEXT BL mean reverting</td>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>PHELIX PL mean reverting</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>ABX BL mean reverting jump diffusion</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>POWERNEXT BL mean reverting jump diffusion</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>PHELIX PL mean reverting jump diffusion</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>ABX BL EVT</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>POWERNEXT BL EVT</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>PHELIX PL EVT</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Source: Author’s analysis

According to backtesting results, we can accept all examined models. The most conservative is extreme value theory. Following table tests model’s risk overestimation as it was defined by equation 2.18:

Table 3.4: Review of risk overestimation figures

<table>
<thead>
<tr>
<th>Model</th>
<th>( \text{VaR}_r )</th>
<th>( \text{CVaR}_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABX BL mean reverting</td>
<td>528.6</td>
<td>603.7</td>
</tr>
<tr>
<td>POWERNEXT BL mean reverting</td>
<td>523.9</td>
<td>596.7</td>
</tr>
<tr>
<td>PHELIX PL mean reverting</td>
<td>653.4</td>
<td>744.1</td>
</tr>
<tr>
<td>ABX BL mean reverting jump diffusion</td>
<td>542.4</td>
<td>641.5</td>
</tr>
<tr>
<td>POWERNEXT BL mean reverting jump diffusion</td>
<td>573.6</td>
<td>676.3</td>
</tr>
<tr>
<td>PHELIX PL mean reverting jump diffusion</td>
<td>713.1</td>
<td>835.9</td>
</tr>
<tr>
<td>ABX BL EVT</td>
<td>387.4</td>
<td>470.8</td>
</tr>
<tr>
<td>POWERNEXT BL EVT</td>
<td>410.6</td>
<td>494.7</td>
</tr>
<tr>
<td>PHELIX PL EVT</td>
<td>539.5</td>
<td>645.0</td>
</tr>
</tbody>
</table>

Source: Author’s analysis

Surprisingly, the most prudent model, due to backtesting results, is the lowest risk overestimating. Considering both examined statistics, the mean reverting model performance is higher compared to the mean reversion jump diffusion model, within selected period and applied on chosen time series.
5. Conclusion

Mean reverting model, jump diffusion mean reverting model and extreme value theory were tested on three selected high volatile electricity spot day ahead time series. VaR and CVaR figures were calculated and backtesting procedure (within period from 1.1.2008 until 31.12.2009) was run in order to assess model’s ability to capture higher volatility and extreme jump occurrence. General concept extreme value theory provided the best performance in context of backtesting with minimal risk overestimating measure.
References


Appendix A – Returns characteristics

Figure A.1: Daily returns and time series characteristics
Appendix B – Unseasonal returns statistics

Figure B.1: Unseasonal returns and time series characteristics
Appendix C – Mean reverting model parameters estimation

Figure C.1: APX BL mean reverting model parameters estimation

Figure C.2: POWERNEXT BL mean reverting model parameters estimation
Figure C.3: PHELIX PL mean reverting model parameters estimation
Appendix D – Mean reverting jump diffusion model parameters estimation

Figure D.1: APX BL mean reverting jump diffusion model parameters estimation

Figure D.2: POWERNEXT BL mean reverting jump diffusion model parameters estimation
Figure D.3: PHELIX PL mean reverting jump diffusion model parameters estimation
Appendix E – Extreme Value Theory parameters estimation

Figure E.3: Extreme Value Theory parameters estimation
Appendix F – Mean reverting model VaR and CVaR figures

Figure F.1: Mean reverting model VaR and CVaR of APX BL

Figure F.2: Mean reverting model VaR and CVaR of POWERNEXT BL
Figure F.3: Mean reverting model VaR and CVaR of PHELIx PL
Appendix G – Mean reverting jump diffusion model VaR an CVaR figures

Figure G.1: Mean reverting model jump diffusion VaR and CVaR of APX BL

Figure G.2: Mean reverting model jump diffusion VaR and CVaR of POWERNEXT BL
Figure G.3: Mean reverting model jump diffusion VaR and CVaR of PHELIX PL
Appendix H – Extreme Value Theory VaR and CVaR figures

Figure H.1: Extreme value theory VaR and CVaR of APX BL

Figure H.2: Extreme value theory VaR and CVaR of POWERNEXT BL
Figure H.3: Extreme value theory VaR and CVaR of PHELIX PL
## Appendix I – Number of CVaR excesses

Table I.1: Number of CVaR excesses

<table>
<thead>
<tr>
<th></th>
<th>CVaR\text{lower_}excess</th>
<th>CVaR\text{upper_}excess</th>
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<tbody>
<tr>
<td>ABX BL mean reverting</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>POWERNEXT BL mean reverting</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>PHELIX PL mean reverting</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>ABX BL mean reverting jump diffusion</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>POWERNEXT BL mean reverting jump diffusion</td>
<td>5</td>
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<tr>
<td>PHELIX PL mean reverting jump diffusion</td>
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<tr>
<td>ABX BL EVT</td>
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<td>PHELIX PL EVT</td>
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</table>

Source: Author's analysis