Financial Crises and Systemic Bank Runs in a Dynamic Model of Banking

Roberto Robatto
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Contents

Abstract 3

1. Introduction 4

2. Model 9

3. Equilibrium 13

4. Results 21

5. Monetary policy 28

6. Conclusions 33
Financial Crises and Systemic Bank Runs in a Dynamic Model of Banking

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Abstract

I present a new dynamic general equilibrium model of banking to analyze monetary policy during financial crises. A novel channel gives rise to multiple equilibria. In the good equilibrium, all banks are solvent. In the bad equilibrium, many banks are insolvent and subject to runs. The bad equilibrium is also characterized by deflation and a flight to liquidity. Some central bank interventions are more effective than others at eliminating the bad equilibrium. Interventions that do not eliminate the bad equilibrium still counteract deflation and reduce the losses of insolvent banks, but, for some parameter values, amplify the flight to liquidity.

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1. Introduction

A peculiar event of the 2007-2009 US financial crisis was a dramatic increase in the private sector’s willingness to hold liquid assets, a “flight to liquidity.” The Federal Reserve reacted aggressively at the time, implementing unconventional monetary policies. The flight to liquidity and the interventions of the Fed resulted in an approximately constant price level and a sizable drop in the money multiplier.\(^1\) The Great Depression saw a similar drop in the money multiplier. Friedman and Schwartz (1963) argue that the absence of adequate Federal Reserve intervention at that time generated deep deflation, making what would otherwise have been a modest or deep recession the Great Depression.

During both these crises, several financial institutions became insolvent and were subject to runs. More than one-fifth of the commercial banks in the US suspended operations during the Great Depression (Friedman and Schwartz, 1963). The collapse of Lehman Brothers in September 2008 was followed by a “run on repo” and on other institutions not covered by deposit insurance, as Gorton and Metrick (2012a,b) document.

Motivated by these events, the first contribution of this paper is to provide a new dynamic general equilibrium model of banking with multiple equilibria. The multiplicity of equilibria is based on a debt-deflation channel similar to Fisher (1933), a novel approach in the panic-based bank runs literature. In the good equilibrium, all banks are solvent. In the bad equilibrium, many banks are insolvent and subject to runs. Distress in the banking sector is associated with deflation, a drop in asset prices, and a flight to liquidity (that is, depositors hold more money and fewer deposits at banks, in comparison to the good equilibrium). In the model, runs and insolvencies are systemic events, in the sense that many financial institutions are subject to distress at the same time. Therefore, the model captures the systemic nature of financial crises.

The second contribution is an application of the model to analyze some of the monetary policies used during the recent US financial crisis. In the model, a central bank can inject money into the economy by either 1) buying assets on the market (asset purchases), or 2) setting up liquidity facilities (in order to provide loans to banks). Using numerical simulations of the model, I show that both asset purchases and loans to banks counteract deflation, reduce the losses of insolvent banks, and if the intervention of the central bank is sufficiently large, eliminate the bad equilibrium. This result is consistent with the Friedman-Schwartz hypothesis regarding the Great Depression. In addition, there are two novel results. First, in some circumstances loans to banks eliminate the bad equilibrium while asset purchases do not. Second, for some parameter values, if a temporary monetary injection does not eliminate the bad equilibrium, it amplifies the flight to liquidity.

In the model, households are subject to uninsurable preference shocks that affect the utility of consumption, similarly to Diamond and Dybvig (1983). There is an exogenous supply of two assets in the economy, fiat money and a productive asset (capital). Trading frictions interact with the timing of preference shocks and create a precautionary demand for money to finance consumption expenditure.

\(^1\) The money multiplier is the ratio of broad monetary aggregates, such as M1 or M2, to the monetary base M0.
Banks offer deposits in order to provide money to households on demand. In the model, banks are unregulated institutions that perform maturity transformation without deposit insurance, similar to commercial banks in the 1930s and to the shadow banking system in recent years.

In the banking sector, two frictions are crucial. First, deposits are nominal, i.e., specified in terms of money. Second, capital held by banks is hit by idiosyncratic shocks (in particular, I will consider one-time unanticipated shocks). For an individual bank, a negative shock destroys some of its capital, while a positive shock increases its stock of capital. Since the shocks are idiosyncratic, the aggregate stock of capital is constant. Crucially, there is asymmetric information about the shocks. Each bank observes its own shock, but it takes time for other banks and households to observe them. Thus, households do not know whether their own bank has been hit by a positive or a negative shock.

In a steady-state with no shocks to banks, deposits overcome the frictions that give rise to the precautionary demand for money. When the one-time unanticipated shocks hit banks, a good equilibrium always exists wherein all banks are solvent (including banks hit by a negative shock) and the banking sector functions normally as in steady-state. A bad equilibrium exists for a large subset of the parameter space. The bad equilibrium lasts one period and then the economy reverts to normal. However, the model can be extended so that the bad equilibrium lasts many periods.

A bad equilibrium is characterized by three features. First, the economy experiences deflation and a drop in the nominal price of capital. Second, a bank hit by a negative shock becomes insolvent; the value of capital drops due to the drop in its nominal price and to the negative idiosyncratic shock, while liabilities (deposits) are in nominal terms and thus constant (debt-deflation). Banks hit by a positive shock remain solvent. Asymmetric information prevents depositors from immediately identifying insolvent banks, but eventually, the insolvency of banks becomes common knowledge and insolvent banks are subject to runs. Third, anticipating the possibility of runs, households hold more money and fewer deposits due to the precautionary demand for money (flight to liquidity).

This scenario is an equilibrium because there is a general equilibrium feedback from the flight to liquidity to the drop in prices. With the flight to liquidity, some money is stored under the mattress for precautionary reasons, therefore less money is in circulation for transactions. As a result, the price level drops because it is proportional to the amount of money used for transactions, an argument related to the quantity theory of money. Because of the drop in the price of consumption goods, the asset that produces such goods (capital) is less valuable, and its price drops as well. Deflation occurs (or, more generally, lower-than-anticipated inflation), because all the money is spent in the pre-crisis period and in the good equilibrium.

As deposits are assets that can be converted easily into money, they are part of broad monetary aggregates such as M1. The drop in deposits is therefore equivalent to a drop in M1 and thus in the money multiplier (due to a constant money supply).

\[ \text{Gorton (2008) emphasizes the uncertainty regarding the identities of the financial institutions that incurred significant losses associated with the housing market during the Great Recession.} \]
Within the category of bad outcomes there are actually multiple bad equilibria, more precisely up to three bad equilibria depending on parameters. One bad equilibrium cannot be extended to multi-period crises. The other two bad equilibria can instead be extended to multi-period crises, and I focus most of my analysis on these two. The multiplicity of bad equilibria arises from a strategic complementarity across depositors. That is, if everybody else reduces deposits at banks, an individual depositor wants to do the same.

The first novel result related to monetary policy is a comparison between the ability of loans to banks and of asset purchases to eliminate the bad equilibria. Both policies are successful if monetary injections target a high enough threshold for the price level. If the central bank does not target the threshold price (perhaps due to considerations not captured by the model such as other welfare costs or legal restrictions) the result is more complicated. Asset purchases do not eliminate the bad equilibria, while a moderate monetary injection implemented using loans to banks is successful. It is, however, crucial that loans to banks have the same seniority as deposits. Thus, losses of insolvent banks are borne not only by depositors, but also by the central bank, so households are willing to hold more deposits. With equal seniority, the central bank suffers losses on loans to banks that go bankrupt, even though just off-equilibrium. Crucially, this policy also eliminates the strategic complementarity that gives rise to multiple bad equilibria.

The second monetary policy result is related to monetary injections that do not eliminate the bad equilibria and that are temporary (i.e., the money supply reverts to the pre-crisis level when the panic ceases). For some parameter values, both loans to banks and asset purchases reduce the equilibrium value of deposits, exacerbating the flight to liquidity. Moreover, if without policy intervention there exists only one bad equilibrium, some monetary injections give rise to a further bad equilibrium, in which the flight to liquidity is exacerbated even more. Due to this endogenous amplification, the conclusions of monetary policy analysis may differ from models in which the flight to liquidity is caused by exogenous shocks to money demand (such as Allen et al., 2013, Christiano et al., 2003, and Diamond and Rajan, 2006). This possible amplification of the flight to liquidity is the result of two counteracting effects. First, monetary policy pushes the equilibrium outcome closer to what would prevail if agents did not panic, stabilizing the economy and reducing the flight to liquidity. Second, money injections increase demand for capital regardless of whether the central bank buys capital directly (asset purchases) or gives loans to banks. In the latter case, demand for capital increases because private banks increase purchases of capital after receiving loans. This higher demand increases the price of capital and thus reduces its return. Since banks invest part of their deposits in capital, the drop in the return on capital implies a drop in the return that banks pay to depositors, thereby making depositors less willing to hold deposits. This second mechanism counteracts the first stabilizing force, and the total effect on the equilibrium value of deposits is ambiguous.

3 Seniority refers to the order of repayment in the event of bankruptcy. Senior debts are repaid first during bankruptcy, while other junior debts are repaid thereafter, if residual funds remain.
1.1. Comparison with the literature

Diamond and Dybvig (1983) formalize the notion of bank runs as panics, using multiplicity of equilibria. There are a number of differences between Diamond and Dybvig (1983) and my work. First, there is only one real asset in Diamond and Dybvig (1983), so it is difficult to use their model to analyze monetary injections. In contrast, my model has a specific role for money.\(^4\) Second, the model of Diamond and Dybvig (1983) has exogenous asset returns, and thus is often interpreted as a partial equilibrium model of one bank. My analysis is instead based on a general equilibrium model with endogenous returns, and runs are systemic events that involve a fraction of the banking system. Third, the run equilibrium in Diamond and Dybvig (1983) relies on a coordination failure with regard to the decision to run-not run. In my model, the bad equilibrium is instead based on a debt-deflation channel, and the coordination failure is based on the decision to fly-not fly to liquidity.

Angeloni and Faia (2013), Ennis and Keister (2003), Gertler and Kiyotaki (2013), Martin et al. (2011), and Mattana and Panetti (2014) combine three-period models of runs with the infinite-horizon formulation of business cycle models. The work of Gertler and Kiyotaki (2013) is closely related to mine; multiple equilibria are due to a general equilibrium channel, and crises are systemic. Differently, they do not include money or information asymmetries, and there are runs on all banks rather than just on a fraction of the banking system as in my model.

The models of Carapella (2012) and Cooper and Corbae (2002) are also closely related to mine. Both present monetary models with multiple equilibria, but their focus is on banking intermediation rather than insurance against liquidity risk and runs. In Carapella (2012), multiplicity arises due to a debt-deflation channel similar to my paper. Policy analysis, however, emphasizes the comparison between monetary injections and deposit insurance, rather than among alternative monetary policy tools. In Cooper and Corbae (2002), multiplicity is related to increasing returns to scale in intermediation, and monetary policy eliminates the bad equilibrium by increasing the growth rate rather than increasing the level of money.

Several studies analyze banks in markets with asymmetric information (see e.g., Freixas and Rochet, 2008), and a recent literature incorporates them into dynamic general equilibrium models (e.g., Bigio, 2012, Boissay et al., 2013, and Martinez-Miera and Suarez, 2012).

The assumptions concerning the structure of trading in my model are very similar to Telyukova and Visschers (2013) and are also analogous to Bianchi and Bigio (2013), Lagos and Wright (2005), and Lucas (1990).

Other papers focus on similar aspects of financial crises and policies as mine, but use alternative models. In Brunnermeier and Sannikov (2011), a shock to financial intermediaries triggers debt deflation; monetary policies can help bank recapitalization, but no financial institution is insolvent. In Caballero and Krishnamurthy (2005, 2008), Knightian uncertainty is responsible for a flight to quality.

\(^4\) Other papers such as Allen et al. (2013) and Diamond and Rajan (2006) include money in models of banking, but their focus is different from panic-based runs.
Krishna-murthy (2010) analyzes the role of policy (including monetary policy) to counteract balance sheet amplification and Knightian uncertainty.

Bank runs and the role of asymmetric information have also been analyzed from an empirical standpoint. For the 2008 financial crisis, runs on the “shadow banking system” are discussed by, e.g., Brunnermeier (2009), Duffie (2010), Gorton and Metrick (2012a,b), and Lucas and Stokey (2011), although the debate about their importance is still open (see Krishnamurthy et al., 2012, and Krishnamurthy and Nagel, 2013). Ivashina and Scharfstein (2010) document runs by borrowers who drew down their credit lines. Bank runs are studied by Friedman and Schwartz (1963) for the Great Depression, and by Gorton (1988) for the national banking era (1863-1914). Asymmetric information about banks in the Great Recession is discussed by, e.g., Gorton (2008), and can be inferred also from indirect evidence. Bernanke (2010) and Armantier et al. (2011) emphasize the stigma associated with borrowing from the discount window. A stigma was also associated with banks that borrowed from the government-established RFC (Reconstruction Finance Corporation) in 1932, according to Friedman and Schwartz (1963). Information asymmetries were also important in nineteenth-century banking panics (Gorton and Mullineaux, 1987).
2. Model

The economy is populated by a unit mass of banks indexed by $b \in B \equiv [0,1]$ and a double continuum of households indexed by $h \in H = [0,1] \times [0,1]$. I use $i \in BUH$ as an index that denotes both households and banks.

Time is discrete, and each period is divided into two parts, day and night. I use capital letters to denote quantities and prices during the day, and lower-case letters to denote quantities and prices at night. Superscripts $h$ and $b$ refer to household $h$ and bank $b$.

Appendix A describes an extension to the model that produces a well-defined steady-state. As the extension does not affect the main results of the paper, I postpone it to the Appendix for simplicity.

2.1 Households and banks

Household $h \in H$ enjoys utility from goods $c^h_t$, consumed at night according to:

$$
E_0 \sum_{t=1}^{\infty} \beta^t \varepsilon^h_t \log c^h_t
$$

Where $\varepsilon^h_t$ is a preference shock realized at the beginning of the night and:

$$
\varepsilon^h_t = \begin{cases} 
\varepsilon > 0 & \text{(impatient)} \quad \text{with probability } \kappa \\
\varepsilon = 0 & \text{(patient)} \quad \text{with probability } 1 - \kappa.
\end{cases}
$$

(1)

The preference shock is private information of household $h$, is i.i.d. over time and across households, and the law of large numbers holds for each subset of $H$ with a continuum of households. I impose the normalization:

$$
E(\varepsilon_t) = 1.
$$

(2)

Therefore, equations (1) and (2) imply $\kappa \bar{\varepsilon} = 1$.

The banking sector is perfectly competitive, and the objective of banks is to maximize profits.

2.2 Assets, trading and shocks to capital

Assets. There are three assets in the economy: capital, money, and deposits. Capital is in fixed supply $\overline{K}$. The supply of money $M_t$ is chosen by the central bank and $M_t = \overline{M}$ for all $t$. A deposit

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5 The assumption $M_t = \overline{M}$ is relaxed in Section 5 in the discussion of monetary policy.
issued by bank $b$ is a claim that is redeemable on demand at bank $b$. The supply of deposits is endogenously determined in equilibrium.

**Markets.** Trading takes place in a day market and in a night market, as represented in Figure 1. During the day, there is a Walrasian market in which households and banks trade capital, money, and deposits. The price of money is normalized to one, and $Q_i$ is the price of one unit of capital. Let $K_i^t$, $M_i^t$, and $D_i^t$ be the amount of capital, money, and deposits that agent $i \in \text{HUB}$ has after leaving the day market. After the day market closes, capital produces output with a linear technology $y(K) = ZK$, $0 < Z < \infty$. Total output $y(K) = ZK$ is the only consumption good in the economy. There is no depreciation.

At night, there is another centralized market in which household $h \in \text{H}$ can buy consumption goods $c_h^t$ at price $p_t$, subject to a cash-in-advance constraint. Capital cannot be traded at night. Let $m_i^t$ and $d_i^t$ be the amount of money and deposits of agent $i \in \text{HUB}$ at the end of the night (to be defined later).

**State variables and shocks to capital.** Each agent $i \in \text{HUB}$ starts the day with a vector of state variables $X_i^t$:

$$X_i^t = \{(K_i^{t-1}, m_i^{t-1}, d_i^{t-1}), \psi_i^t\}$$

where $K_i^{t-1}$ is capital, $m_i^{t-1}$ is money, and $d_i^{t-1}$ are deposits whose values have been determined at $t-1$. The initial stock of capital of agent $i$ is $K_i^{t-1}(1 + \psi_i^{t})$ where $\psi_i^{t}$ is an idiosyncratic shock realized at time $t$ with support $\psi_i^t \in \{-\bar{\psi}, 0, \bar{\psi}\}$.

---

6 Households cannot consume output produced by their own stock of capital, similarly to standard models with a cash-in-advance constraint such as Lucas and Stokey (1987). Appendix B.1 provides more discussion about the cash-in-advance constraint assumption.
\(-1 < \psi_i < 0 < \overline{\psi} \). The value of \( \psi_i \) is private information of agent \( i \) during the day, while it becomes common knowledge at night.

The shocks \( \{\psi_i\} \) are idiosyncratic in the sense that the law of large numbers holds: \( \int K_t^i \psi_i^i di = 0 \) I also assume that the law of large numbers holds both within the banking sector and within the household sector:

\[
\int_{\mathcal{B}} K_{t-1}^b \psi_t^b dB = 0, \quad \int_{\mathcal{H}} K_{t-1}^h \psi_t^h Dh = 0.
\]  

(3)

I assume that Pr (\( \psi_i^d = 0 \) for all \( i \in H) = 1, and I will analyze the effects of one-time unanticipated shocks. When the one-time unanticipated shocks hit the economy, shocks take the values \( \psi_i^d = \psi_i \) (negative shock) with probability \( a \epsilon (0, 1) \) and \( \psi_i^d = \overline{\psi} \) (positive shock) with probability \( 1 - a \).\(^7\)

2.3 Banking

Deposits (day). I impose a particular demand-deposit contract, rather than deriving it from an explicit contracting problem. In this Section, I describe the restrictions that I impose on the deposit contract. In Appendix B.3, I provide some justification for these restrictions.

Assumption 2.1. (Nominal demand-deposit contract) A deposit is redeemable for a value specified in terms of money.

Assumption 2.2. (One bank per household) Each household \( h \in H \) can hold \( d \)-posits \( D_t^h \) (at most) at one bank.

Assumption 2.1 imposes that households and banks can use only demand-deposit contracts in nominal terms. Assumption 2.2 can be justified by costs of maintaining banking relationships. Formally, the cost would be zero if household \( h \) holds deposits at one bank, and infinite if household \( h \) holds deposits at two or more banks. Assumption 2.2 can be relaxed, but it is crucial that households cannot hold deposits at a large number of banks.

To clarify the notation and the timing, note that household \( h \) starts period \( t \) with preexisting deposits \( d_{t-1}^h \), and bank \( b \) starts period \( t \) with preexisting deposits \( d_{t-1}^b \). The choice \( D_t^h \) taken by household \( h \) is thus a decision regarding rolling over her preexisting deposits \( d_{t-1}^h \) (fully or partially) and/or increasing her deposits. For instance, if \( D_t^h = d_t^h \), then the value of deposits of household \( h \) stays constant. For bank \( b \), the difference \( D_t^b - d_t^b \) is the net issuance of deposits. If \( D_t^h > d_t^h \), bank \( b \) increases its

\(^7\) Appendix B.2 provides an interpretation of the shock \( \psi_i \) and presents an alternative formulation to define the shocks.
deposits and thus receives new resources from households. Otherwise, bank $b$ reduces its amount of preexisting deposits and must pay back some resources to households.\footnote{To describe precisely the interaction between banks and depositors, I must specify what happens if many preexisting deposits are not rolled over during the day and the bank does not have enough resources to repay them, i.e., there is not enough preexisting money $m_{h,t-1}$ and capital $K_{t-1}^h (1 + \psi_t^h)$. If such circumstances occur, the bank is shut down immediately and depositors get pro-rata repayments.}

For future reference, let $H(b) \subseteq H$ be the set of depositors of bank $b \in B$, and let $b(h) \in B$ be the bank of household $h \in H$.

**Withdrawals (night).** At night, households learn the realization of their own preference shock $\xi_t^h$. They then decide to withdraw $w^h_t$ from their own bank subject to a sequential service constraint. They then decide to consume $c^h_t$.

**Assumption 2.3.** (Withdrawals) As long as bank $b(h) \in B$ has money to pay withdrawals, depositor $h$ can withdraw any amount of money less or equal than the value of her deposits chosen during the day, $D^h_t$.

In the event of large withdrawals from a bank, the bank might not have enough cash to serve all households. Household $h$ can withdraw any amount $0 \leq w^h_t \leq \min \{D^h_t, l^h_t \}$ where $l^h_t \in \{0, + \infty\}$ is a limit on withdrawals determined by the position in the line. If household $h$ is served when the bank is out of money, then $l^h_t = 0$ and thus $w^h_t = 0$. If household $h$ is served when the bank still has money, then $l^h_t = + \infty$ and $0 \leq w^h_t \leq D^h_t$.

Bank $b$ is subject to a run if the limit on withdrawals is $l^h_t = 0$ for some $h \in H(b)$. If bank $b$ is subject to a run, the bank is liquidated at $t + 1$ while the day market is open. Liquidation works as follows. All assets of the bank are sold on the market, and deposits not withdrawn at night are repaid (if the value of assets is insufficient, depositors are repaid pro-rata).\footnote{The case in which the value of assets of the banks is higher then the value of deposits not withdrawn is discussed for completeness in Appendix A, but it is not relevant for the main results of the paper.}

Due to the cash-in-advance constraint, consumption expenditures $p_t c^h_t$ cannot exceed the sum of money $M^h_t$ chosen during the day and withdrawals $w^h_t$, chosen at night, $p_t c^h_t \leq M^h_t + w^h_t$.

Banks do not make any economic decisions at night. The amount of money withdrawn by depositors of bank $b$ is $w^b_t = \int_{H(b)} w^h_t dh$. Withdrawals $w^b_t$ are limited by the feasibility constraint $w^b_t \leq M^b_t$ (money that is distributed at night to depositors cannot exceed the amount $M^b_t$ that bank $b$ held at the end of the day).

**Return on deposits.** During the day of period $t$, banks promise to pay a return $1 + R^D_t$ (in $t + 1$) on deposits that are not withdrawn that night.\footnote{The term $1 + R^D_t$ is the face value of deposits, conditional on not withdrawing at night. Note that $R^D_t$ is a market price that is taken as given by both banks and households. The results are unchanged if I allow each bank to post a bank-specific return during the day.}

**Assumption 2.4.** (Return on deposits withdrawn) Banks pay no return on deposits withdrawn at night.
Banks might not have enough resources to pay the promised return \( R^D_t \). Define \( \hat{r}_t \leq R^D_t \) to be the actual return on deposits. Note that \( \hat{r}_t \) can be lower than the promised return; if that is the case, then the quantity \( 1 + \hat{r}_t \) has the interpretation of recovery rate. The value of deposits at the end of the night \( d_t^h \) for household \( h \) is \( d_t^h \equiv (D_t^h - w_t^h) \left( 1 + \hat{r}_t^{(h)} \right) \). For bank \( b \in B \), it is useful to define the value of deposits at the end of the night as \( d_t^b \equiv (D_t^b - w_t^b) \left( 1 + R_t^D \right) \). That is, the value depends on the promised return \( R^D_t \), rather than the actual return \( \hat{r}_t \).

### 2.4 State of the economy and sunspot

The aggregate state of the economy \( X_t \) at the beginning of the day is \( X_t = \{ Pr^B_t, s_t \} \), where \( Pr^B_t \) is the probability distribution over the states of banks \( X_b^t \) and \( s_t \) is a sunspot.\(^{11}\) The sunspot is an exogenous process that determines equilibrium selection, when multiple equilibria exist. The sunspot \( s_t \) selects the good equilibrium with probability one, so the bad equilibrium is unanticipated. For all agents in the model, knowledge of the aggregate state only conveys information about the overall distribution of assets and liabilities of banks. It does not clarify the assets, liabilities, and realization of \( \psi_t^b \) of any particular bank \( b \in B \).

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\(^{11}\) I will impose restrictions on initial conditions so that the states of banks take finitely many values.
3. Equilibrium

I describe the problem of banks (Section 3.1), the problem of households (Section 3.2) and then I define the notion of equilibrium (Section 3.4). For future reference, let $R^K_t$ be the nominal return on capital:

$$1 + R^K_t = \frac{Q_{t+1} + Zp_t}{Q_t}$$  \hspace{1cm} (4)

3.1 Bank problem

Given the vector of state variables $X^b_t = \{(K^b_{t-1}, m^b_{t-1}, d^b_{t-1}, \psi^b_t)\}$ and the price of capital $Q$, the balance sheet of a bank $b$ at the beginning of the day is:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of capital = $K^b_{t-1} (1 + \psi^b_t) Q_t$</td>
<td>Value of deposits = $d^b_{t-1}$</td>
</tr>
<tr>
<td>Money = $m^b_{t-1}$</td>
<td>Net worth = $N^b_t$</td>
</tr>
</tbody>
</table>

where net worth is the difference between the value of assets and the value of deposits:

$$N^b_t \equiv K^b_{t-1} (1 + \psi^b_t) Q_t + m^b_{t-1} - d^b_{t-1}.$$  \hspace{1cm} (5)

The net worth $N^b_t \in \mathbb{R}$, so it can be either positive or negative. If $N^b_t \geq 0$, the bank is solvent (the value of its assets is larger than deposits $d^b_{t-1}$). If $N^b_t < 0$, the bank is insolvent (the value of its assets is less than deposits $d^b_{t-1}$). Note that a bank with negative net worth can be active in equilibrium because of asymmetric information about $\psi^b_t$. Since bank $b$ takes the price $Q$ as given, the net worth $N^b_t$ summarizes the vector of state variables $X^b_t$ for the purpose of understanding the choices of bank $b$. Profit maximization is equivalent to maximizing $N^b_{t+1}$ subject to limited liability, $\max \{0, N^b_{t+1}\}$ (see Appendix A.2 for more details). Given $N^b_t$, bank $b \in B$ chooses deposits $D_t^b$, money $M_t^b$, and capital $K_t^b$, taking as given the market return on deposits $R^D_t$, and withdrawals $w_t^b$ by depositors at night:

$$\max_{D^b_t, M^b_t, K^b_t} \mathbb{E}_{\psi_t} \max \{0, N^b_{t+1}\}$$  \hspace{1cm} (6)

subject to the budget constraint (7) and the law of motion of worth (8):

$$K^b_t Q_t + M^b_t \leq D^b_t + N^b_t$$  \hspace{1cm} (7)
where $m^b_t$ and $d^b_t$ are money and deposits at the end of the night of time $t$.

\begin{equation}
N^b_{t+1} = K^b_t (1 + \psi^b_{t+1}) Q_{t+1} + m^b_t - d^b_t \tag{8}
\end{equation}

The expectation $\mathbb{E}_\psi$, is taken with respect to the shock to capital $\psi_{t+1}$. Banks must also satisfy non-negativity constraints $D^b_t > 0$, $M^b_t > 0$, and $K^b_t > 0$. The solution to the problem of banks is summarized by Proposition 3.1 when the non-negativity constraints are not binding, which is relevant for most of the paper; the proof is provided in Appendix C. Appendix D presents and discusses a situation in which the non-negativity constraints are binding.

**Proposition 3.1.** Given $N^b_t$ and prices $Q_t$, $R^K_t$, $R^D_t \geq 0$, the optimal choice of bank $b$ is:

1. deposits:
   \begin{equation}
   D^b_t = \begin{cases} 
   0 & \text{if } R^D_t > R^K_t \\
   \text{any amount} & \text{if } R^D_t = R^K_t \\
   \infty & \text{if } R^D_t < R^K_t
   \end{cases} \tag{9}
   \end{equation}

2. money holding $M^b_t = \kappa D^b_t$,

3. capital holding $K^b_t = \frac{N^b_t + D^b_t - M^b_t}{Q_t}$, provided that the non-negativity constraints $M^b_t > 0$ and $K^b_t > 0$ are not binding.

To understand the result, consider first a bank that starts with zero net worth, $N^b_t = 0$. The law of large numbers about the preference shocks $\varepsilon^h_t$ implies that a fraction $k$ of depositors withdraw at night to finance consumption expenditures. Thus banks keep an amount of money $M^b_t = kD^b_t$ that is just enough to finance such withdrawals. The remaining resources $D^b_t - M^b_t = (1 - k) D^b_t$ are invested in capital, yielding a net return $(1 - k) D^b_t R^K_t$ in $t + 1$. As the bank will have to pay the return $(1 - k) D^b_t R^D_t$ on deposits not withdrawn, the profit of the bank is $(1 - k) D^b_t R^K_t$. Thus, the bank chooses $D^b_t = 0$ if $R^K_t < R^D_t$ (otherwise it would make negative profit), $D^b_t = \infty$ if $R^K_t > R^D_t$ (because it can make strictly positive profits on every dollar of deposit), and it is indifferent among any $D^b_t$ if $R^K_t = R^D_t$ (making zero profits).

If a bank has a positive net worth, $N^b_t > 0$, a similar analysis applies. The bank invests a fraction $k$ of deposits in money and a fraction $1 - k$ in capital. The whole net worth $N^b_t$ is invested in capital to maximize the value of net worth tomorrow.

For a bank with negative net worth, $N^b_t < 0$, I explain here only the relevant equilibrium case $R^K_t = R^D_t$.

A bank $b$ with negative net worth does not earn profits on deposits if $R^K_t = R^D_t$. Therefore, its net worth
at $t + 1$ remains negative. Consequently, the bank is indifferent between its choices (because its payoff will always be zero due to limited liability) and taking the same choices as a solvent bank is (weakly) optimal.

### 3.1.1 Actual return on deposits

The actual return on deposits $r^b_t$ is defined as:

$$r^b_t \equiv \min \left\{ R^D_t, \tilde{r}^b_t \right\}.$$  \hfill (10)

The variable $\tilde{r}^b_t$ is the return that can be paid to deposits not withdrawn using proceeds from selling output $ZK^b_t$ and the value of capital $K^b_tQ_t$, that a bank has at the beginning of $t + 1$. Thus, $\tilde{r}^b_t$ solves:

$$\mathbb{E}_\psi \left\{ K^b_t \left( 1 + \psi^b_{t+1} \right) Q_{t+1} \right\} + ZK^b_t p_t = (D^b_t - w^b_t) \left( 1 + \tilde{r}^b_t \right)$$

or, using $\psi^b_{t+1} = 0$ with probability one and rearranging:

$$1 + \tilde{r}^b_t = \frac{K^b_t (Q_{t+1} + Z p_t)}{D^b_t - w^b_t}.$$

### 3.1.2 Fraction of depositors served during a run

If all depositors of bank $b$ attempt to withdraw money at night, only a fraction $f^b_t$ of depositors will be served. The fraction of depositors served is:

$$f^b_t = \frac{M^b_t}{D^b_t}$$  \hfill (12)

From the viewpoint of household $h$ that has deposits at bank $b(h)$, if all depositors of bank $b$ attempt to withdraw their deposits, then household $h$ is able to withdraw with probability $f^b_t(h)$.

### 3.2 Household problem

Given the vector of state variables $X^h_t = \{(K^h_{t-1}, m^h_{t-1}, d^h_{t-1}), \psi^h_t\}$ of household $h$ and the price of capital $Q_t$, the nominal wealth $A^h_t$ of household $h$ is:

$$A^h_t = K^h_{t-1} \left( 1 + \psi^h_t \right) Q_t + m^h_{t-1} + d^h_{t-1}.$$  \hfill (13)

---

12 Note also that the bank cannot invest 100% of its deposits in money because $M^b < D^b_t + N^b$ using the budget constraint (7) and $N^b < 0$. 

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**WORKING PAPER SERIES N. 62 - DECEMBER 2014**

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Household $h \in H$ is assigned a bank $b(h) \in B$. Let $n^h_t = \{z^h_t, r^h_t, l^h_t\} \in \mathcal{N}$ be the vector of variables whose value is learnt by household $h$ at night, where $\mathcal{N} = \{ n = \{\varepsilon, r, l\} | \varepsilon \in \{\varepsilon, \varepsilon\}, r \in \mathbb{R}, l \in \{0, +\infty\}\}$. First, household $h$ forms $P^h_t (r^h_t = r, l^h_t = l)$ that, combined with the exogenous process for $\varepsilon^h_t$ described in (1), imply a probability distribution over $n \in \mathcal{N}$. Second, during the day, household $h$ chooses money $M^h_t$, deposits $D^h_t$, and capital $K^h_t$. Third, at night, household $h$ observes $n^h_t$ and chooses withdrawals $w^h_t(n^h_t)$ and consumption $c^h_t(n^h_t)$.

Let $V(A^h_t)$ be the value of holding nominal wealth $A^h_t$. The Bellman equation is:

$$V_t(A^h_t) = \max_{M^h_t, D^h_t, K^h_t} \mathbb{E}_{\varepsilon} \left\{ \max_{w^h_t(n^h_t), c^h_t(n^h_t)} \left[ \varepsilon^h_t \log c^h_t(n^h_t) + \beta \mathbb{E}_{\psi} V_{t+1} \left( A^h_{t+1}(n^h_{t+1}, \psi^h_{t+1}) \right) \right] \right\}$$

subject to the budget constraint (15), the limit on withdrawals (16), the cash-in-advance constraint (17), and a non-negativity constraint on money $M^h_t > 0$, deposits $D^h_t > 0$, and capital $K^h_t > 0$:

$$M^h_t + D^h_t + Q_t K^h_t \leq A^h_t$$  \hspace{1cm} (15)

$$0 \leq w^h_t(n^h_t) \leq \min \{ D^h_t, l^h_t \}$$  \hspace{1cm} (16)

$$p_t c^h_t(n^h_t) \leq M^h_t + w^h_t(n^h_t)$$  \hspace{1cm} (17)

where the value of wealth $A^h_{t+1}(n^h_t, \psi^h_{t+1})$ is:

$$A^h_{t+1}(n^h_t, \psi^h_{t+1}) = \left[ K^h_t \left( 1 + \psi^h_{t+1} \right) \right] Q_{t+1} + d^h_t(n^h_t) + m^h_t(n^h_t)$$  \hspace{1cm} (18)

and:

$$d^h_t(n^h_t) = [D^h_t - w^h_t(n^h_t)] \left( 1 + r^h_t \right)$$  \hspace{1cm} (19)

$$m^h_t(n^h_t) = [M^h_t + w^h_t(n^h_t) - p_t c^h_t(n^h_t)] + p_t Z K^h_t.$$  \hspace{1cm} (20)

The term $d^h_t(n^h_t)$ represents deposits not withdrawn $D^h_t$ - $w^h_t(n^h_t)$ plus the actual return $r^h_t$ paid by bank $b(h)$. The term $m^h_t(n^h_t)$ is money at the end of the night, which is the sum of the unspent money at night (i.e., money held during the day $M^h_t$, plus withdrawals $w^h_t(n^h_t)$ minus consumption expenditure $c^h_t(n^h_t)$ plus proceeds from selling output $Z K^h_t$ at night at price $p_t$. The expectation $\mathbb{E}_{\varepsilon}$ is taken with respect to the beliefs over $n \in \mathcal{N}$, and the expectation $\mathbb{E}_{\psi}$ is taken with respect to the shock to capital $\psi^h_{t+1}$.

If $R^h_t = R^h_t$ and household $h$ has belief $\rho^h_t = R^h_t$, with probability one (which is the case in the good equilibrium), households are indifferent between investing directly a fraction of their wealth in capital, or depositing more and letting banks buy capital on their behalf. To simplify the derivation, I impose Assumption 3.2.
Assumption 3.2. If household \( h \in H \) is indifferent among several choices of \( D_t^h \), the household selects the smallest \( D_t^h \) that maximizes her utility.

Assumption 3.2 implies that households use banks only to insure against liquidity risk, and invest directly in capital all the wealth they want to carry to \( t + 1 \). The assumption is irrelevant for the bad equilibrium (because the optimal \( D_t^h \) is unique), and it does not affect prices in the good equilibrium.

Proposition 3.3 states the solution to problem (14), focusing on the relevant case \( R_t^D = R_t^K \). The proof is provided in Appendix C.

Proposition 3.3. Given beliefs \( \Pr_t^h(\cdot) \) and prices \( Q_t, R_t^K, \) and \( R_t^D = R_t^K, \) household \( h \) chooses:

- (day) \( M_t^h = \eta_t^M A_t^h, \quad D_t^h = \eta_t^D A_t^h, \quad Q_t K_t^h = \eta_t^K A_t^h, \) where \( \eta_t^M, \eta_t^D, \eta_t^K \in [0, 1] \) are independent of \( A_t^h \) and \( \eta_t^M + \eta_t^D + \eta_t^K = 1; \)
- (night) withdrawals and consumption:

\[
w_t^h = w^h(n_t^h) = \begin{cases} 
D_t^h & \text{if } z_t^h = \bar{z}, \quad r_t^{b(h)} \in \mathbb{R}, \quad \text{and } l_t^h = +\infty \\
0 & \text{if } z_t^h = \bar{z}, \quad r_t^{b(h)} \in \mathbb{R}, \quad \text{and } l_t^h = 0 \\
D_t^h & \text{if } z_t^h = 0, \quad r_t^{b(h)} < 0, \quad \text{and } l_t^h = +\infty \\
0 & \text{if } z_t^h = 0, \quad r_t^{b(h)} < 0, \quad \text{and } l_t^h = 0 \\
0 & \text{if } z_t^h = 0, \quad r_t^{b(h)} \geq 0, \quad \text{and } l_t^h \in \{0, +\infty\}, \\
0 & \text{if } c_t^h = 0 \\
\frac{M_t^h + w^h(n_t^h)}{p_t} & \text{if } c_t^h = \bar{c} \\
\end{cases}
\]

Since the felicity from consumption is log, I guess and verify that household choices during the day are proportional to initial wealth \( A_t^h \). At night, an impatient household \( (c_t^h = \bar{c}) \) withdraws deposits if unconstrained \( (l_t^h = +\infty) \) and uses money \( M_t^h = \eta_t^M A_t^h \) and withdrawals \( w_t^h \) to finance her consumption expenditures. If the household is patient \( (c_t^h = 0) \), her choice of consumption is zero, but she is nonetheless willing to withdraw if the actual return on deposits is negative \( (r^{b(h)} < 0) \). In this crucial case, the nominal return on money is zero, thus higher than the nominal return on deposits not withdrawn. The household runs on the bank and withdraws all the available deposits \( D_t^h \), if the bank still has money while household \( h \) is served \( (l_t^h = +\infty) \). If instead the bank has no money \( (l_t^h = 0) \), the household is stuck with zero withdrawals and receives a negative return on deposits.

Since \( M_t^h, D_t^h, \) and \( K_t^h \) are proportional to initial wealth \( A_t^h \), Corollary 3.4 holds.

Corollary 3.4. The choices \( \{M_t^h, D_t^h, K_t^h\}_{h \in H} \) of the household sector can be described by a representative household with initial wealth \( \bar{A}_t \)

Consequently, the shocks \( \psi_t^h \) to the capital owned by the household sector are irrelevant from an equilibrium perspective, because they simply modify the distribution of wealth, but they do not...
influence the total value of wealth $\bar{A}_t$. It is, however, crucial that the idiosyncratic shocks to capital hit the balance sheet of banks, creating heterogeneity in the banking sector.

3. 3 Market clearing conditions

The market clearing conditions are as follows.

\[
\text{Capital market, day: } \int_B K^b_t db + \int_H K^h_t dh = \bar{K}. \quad (21)
\]

\[
\text{Money market, day: } \int_B M^b_t db + \int_H M^h_t dh = \bar{M}. \quad (22)
\]

\[
\text{Deposits, day: } \int_B D^b_t db = \int_H D^h_t dh. \quad (23)
\]

\[
\text{Goods market, night: } \int_H c^h_t dh = Z\bar{K}. \quad (24)
\]

3. 4 Equilibrium definition

Given the state of the economy $X_t$ (described in Section 2.4), the distribution over banks’ state $P^{B}_t$, and the price of capital $Q_t$ imply the distribution $P^{N}_t$, over net worth $\{N^b_t\}_{b \in B}$ defined by:

\[
P^{N}_t (N^b_t = N; Q_t) = \sum_{\{X^b_t[K^b_{t-1}(1+\psi^b_t)Q_t+m^b_{t-1}-d^b_{t-1}=N]\}} P^{B}_t (X^b_t).
\]

Although the probability $P^{B}_t$ over $X^b_t$ is given by the state of the economy, the probability $P^{N}_t$, is an endogenous object because it depends on the price of capital $Q_t$. For a given $P^{B}_t$, the price of capital influences the solvency of banks in the economy. The role of $Q_t$ in the determination of net worth is central to the existence of multiple equilibria.

I claim that, given prices, a link between $P^{N}_t$, and the distribution over $\left\{r^b_t(h), l^h_t\right\}_{h \in H}$ can be established. The net worth $N^b_t$ of bank $b \in B$ determines the choices of money $M^b_t$, deposits $D^b_t$, and capital $K^b_t$ (see Proposition 3.1), which in turn determine $r^b_t$ and $l^h_t$ (see equations (10) and (12)).

\[
\text{I denote: } Pr_t^{(r, l)} \left( r^b_t(h) = r, \quad l^h_t = l \right), \quad r \in \mathbb{R} \text{ and } l \in \{0, +\infty\} \quad (25)
\]

to be the probability distribution over the actual return on deposits $r^b_t(h)$ and the limit on withdrawal $l^h_t$ for household $h \in H$. In equilibrium, I require households’ beliefs $P^{f}_t$, to be rational, in the sense that they must be equal to the realized probability distribution (25).

\[\text{13} \quad \text{Recall from Section 3.1.2 that an household is served \(f^h_t = +\infty\) with probability \(f^{b(h)}_t\) if all depositors run on bank \(b\).} \]
Note that, since I force $R^D_t$ to be equalized across all banks, I impose a pooling equilibrium in the banking market, similar to Akerlof (1970). The results are unchanged if I allow each bank $b$ to post a bank-specific promised return on deposits. In this case, the equilibrium that arises is still a pooling one because bad banks want to imitate good banks to survive as long as possible.

The next definition formalizes the equilibrium concept.

**Definition 3.5.** Given the initial state of the economy $X_0$, an equilibrium is a collection of:

- prices $Q_t$ and $p_t$, and return on capital $R^K_t$, and on deposits $R^D_t$;
- household beliefs $P^h_t(\cdot)$, for all $h \in H$;
- Household choices $\{M^h_t, M^h_t, K^h_t, \{w^h(n^h_t), c^h(n^h_t)\}_a^N\}$ for all $h \in H$;
- bank choices $\{D^b_t, M^b_t, K^b_t\}$ for all $b \in B$;
- limits on withdrawals $l^b_t \in (0, +\infty]$ for all $h \in H$;
- actual return on deposits $R^b_t$ and fraction of depositors served in the event of a run $f^b_t$, for all $b \in B$;
- such that:

  - (banks: optimality, returns, and limits on withdrawals) banks solve problem (6); $r^b_t$ and $f^b_t$ satisfy, respectively, equations (10) and (12) and:

\[
\int_{\mathbb{H}(b)} w^h(n^h_t) l^h_t = +\infty \; dh > M^b_t;
\]

  - (households’ optimality) households solve problem (14) and Assumption 3.2 holds;

  - (rational expectations) households’ beliefs are rational, i.e., for all $h \in H$

\[
P^h_t(r^h_{l}, l^h_t = l) = P^h_t(r^h_{l} = r, l^h_t = l), \quad r \in \mathbb{R} \text{ and } l \in \{0, +\infty\};
\]

  - (market clearing) the market clearing conditions hold.

I focus on symmetric equilibria in which banks with the same net worth make the same choices, in particular for deposits $N^b_t = N^b_{t'} \Rightarrow D^b_t = D^b_{t'}$ for $b, b' \in B$.

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14 This condition says that if a household faces a limit on withdrawals $l^h_t = 0$, then the unconstrained amount of withdrawals $\int_{\mathbb{H}(b)} w^h(n^h_t) l^h_t = +\infty$ $dh$ is not feasible).
4. Results

Section 4.1 first describes the steady-state with no shocks to capital. Then, starting from the economy in steady-state, I consider the effects of one-time unanticipated idiosyncratic shocks to capital at time \( t \), \( \psi_t \in \{ \nu_t, \overline{\nu}_t \} \) for all \( i \in \text{HUB} \). At time \( t \), multiple equilibria can arise: a good equilibrium where prices and aggregate quantities are the same as in the steady-state; and up to three bad equilibria described in Section 4.2. If the economy experiences a crisis at time \( t \) (bad equilibrium), the crisis lasts one period, and the economy is in steady-state from \( t + 1 \) onward.

I impose two restrictions on initial conditions. All banks are alike at the beginning of the day, and their holdings of capital and money are enough to guarantee that banks hit by the negative shock \( \nu_t \) are solvent in the good equilibrium.

Assumption 4.1. At time \( t \), the vector of state variables \( X_t^b \) is the same for all banks, i.e., for all \( b \), \( b \in \text{B} \), and they satisfy:

\[
K_{t-1}^b (1 + \psi_t) \left[ \frac{\beta}{1 - \beta} + \left( \frac{1}{\kappa} - 1 \right) \right] \frac{M_t}{K} + m_{t-1}^b - d_{t-1}^b \geq 0.
\]

4.1 Steady-state and good equilibrium

Steady-state.\(^{15}\) In steady-state, prices are constant (\( Q_t = Q^* \), \( p_t = p^* \) and \( R^*_K = R^*_p \)). Due to Assumption 4.1, all banks are identical and solvent. The return on deposits is equal to the return on capital (\( R^*_D = R^*_K = R^*_p \)); all banks pay the promised return on deposits that are not withdrawn (\( R^*_D = R^*_K = R^*_p \)); and there are no runs (\( R^*_h = +\infty \) for all \( h \)). Therefore, banks pool the liquidity risk of households, insuring them against preference shocks. The representative household holds deposits \( D^* \) and no money (\( M^*_h = 0 \)) because the well-functioning banking system offsets the precautionary demand for money. Withdrawals at night are used to finance consumption expenditure. Appendix A.4 presents the complete characterization of \( Q^*, p^*, R^*, \) and \( D^* \) in closed form, as functions of the parameters.

Good equilibrium. If idiosyncratic shocks to capital \( \psi_t \in \{ \nu_t, \overline{\nu}_t \} \) hit the economy, equation (26) guarantees that a good equilibrium exists. The idiosyncratic shocks imply a redistribution of capital within the banking sector and within the household sector, but prices and aggregate quantities in the good equilibrium are the same as in the steady-state. Intuitively, since the shocks are idiosyncratic, they have no effects on aggregate variables, because all banks remain solvent (due to Assumption 4.1). The characterization of the good equilibrium is provided in Appendix A.4.

\(^{15}\) The existence of a well-defined steady-state requires an extension described in Appendix A.
4.2 Bad equilibria

When unanticipated shocks to capital $\psi$ and $\bar{\psi}$ hit the economy, the good equilibrium is not the unique one, for a large subset of the parameter space. There can be up to three bad equilibria, depending on parameters.

In one of the bad equilibria, all banks are insolvent ($N_t^b < 0$ for all $b$), and the economy experiences a “bankless crisis”. No bank is active at time $t$, so households hold money to self-insure against the preference shocks. As all banks are insolvent, asymmetric information is irrelevant in this equilibrium. In $t + 1$, new banks are active and the economy reverts to normal. The bankless crisis cannot be extended to result in a multi-period crisis. This bad equilibrium is discussed in Appendix D.

Henceforth, I focus on the other two bad equilibria, in which only a fraction of banks in the economy are subject to runs and asymmetric information is crucial. These bad equilibria can be extended to result in multi-period crises. Some discussion about multi-period crises is provided in Appendix B.4, but the actual analysis is left for future research.

The channel that gives rise to bad equilibria. Since all banks are alike in $t - 1$, and the shock to capital can take only two values, $\psi_t^b \in \{\psi, \bar{\psi}\}$, in equilibrium there are two groups of banks. I will use $N_t(\psi)$ and $r_t(\psi)$ to denote the net worth and the actual return on deposits of bank $b$ hit by shock $\psi_t^b = \psi$, and similarly $N_t(\bar{\psi})$ and $r_t(\bar{\psi})$ for a bank hit by $\bar{\psi}$.

A bad equilibrium at time $t$ is characterized by four features.

1. The price level is $p_t < p^*$, and the nominal price of capital is $Q_t < Q^*$. The economy experiences deflation and a drop in (nominal) asset prices.

2. Banks hit by the bad idiosyncratic shock $\psi < 0$ are insolvent, $N_t(\psi) < 0$. Banks hit by $\bar{\psi}$ are solvent, $N_t(\bar{\psi}) > 0$.

3. Insolvent banks pay a negative actual return on deposits, $r_t(\psi) < 0 < R_D^b$, and are subject to runs at night. Solvent banks pay the promised return $R_D^b > 0$ and are not subject to runs.

4. The representative household holds deposits $D_h^t < D^*$ and money $M_h^t > 0$ (flight to liquidity).

The insolvency of banks hit by $\bar{\psi}$ (Item 2) is a direct consequence of the drop in $Q_t$ (Item 1). Recall that deposits are expressed in terms of money, so the nominal value of the liabilities of banks is not affected by prices. Insolvent banks pay the actual return on deposits $r_t(\psi) < R_D^b$ (Item 3) because they are insolvent and thus do not have enough resources. Such banks are subject to runs because the actual return on deposits is negative, $r_t(\psi) < 0$, while the return from withdrawing and holding money is zero. Therefore, running is the optimal choice of depositors. The flight to liquidity (Item 4) is a result of

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16 In addition, the existence of the bankless crisis equilibrium does not hinge on the shocks and hitting the economy. In fact, if all banks are alike and have “sufficiently low” net worth, the bad bankless equilibrium exists; see Appendix D.
fear of runs (Item 3). Anticipating runs, households hold more money and fewer deposits at banks, in order to (partially) self-insure against liquidity needs.

The scenario described in Items 1 - 4 is an equilibrium because there is a feedback from the flight to liquidity (Item 4) to the drop of prices (Item 1). With the flight to money by all households, some money is held by households whose realized preference shock is \(c^h_t = 0\). Such money is unspent and stored under the mattresses, so less money is used for transactions in the economy. Multiplying both sides of the goods market clearing condition, equation (24), by \(p_t\):

\[
p_t \int_{\bar{h}}^{h_t} c^h_t dh = p_t \overline{ZK}.
\]

In the good equilibrium, the left-hand side is equal to \(\bar{M}\) because all the money is spent, so \(p_t = p^* = \bar{M} / (\bar{ZK})\). In the bad equilibrium, the left-hand side is smaller than \(\bar{M}\) because some money in the economy is unspent; therefore \(p_t < p^*\). Finally, the real price of capital \(Q_t/p_t\) must be constant because the bad equilibrium does not influence the productivity of capital. Thus, a drop in \(p_t\) is associated with a drop in \(Q_t\). \(^{17}\)

**Welfare.** There is a welfare loss in the bad equilibria due to consumption misallocation across households. Consider households with the same initial wealth. Optimality requires the same level of consumption for these households. Some households are last in line during a run \(l_t = 0\), however, and thus their consumption expenditure is limited by the inability to withdraw money from their own banks. Other households are first in line during runs or face no runs on their own bank \(l_t = +\infty\), so they can withdraw money from their own banks and their consumption expenditure is higher. \(^{18}\)

**Solution method.** I cannot solve for the bad equilibria in closed form, so I compute them numerically using the full non-linear model. I conjecture that, at night, households run on banks hit by the shock \(\bar{\psi}\). Under this conjecture, I solve for a “candidate bad equilibrium” by solving the system of non-linear equations described in Appendix E, using an approach based on the numerical computation of Gröbner bases. \(^{19}\) The “candidate bad equilibrium” is an equilibrium if the initial conjecture \(r_t(\bar{\psi})\) is verified, so that running is indeed optimal for households.

\(^{17}\) The real price of capital is almost constant (but not exactly constant) due to a small wealth effect. In a version of the model with preferences that eliminate wealth effects (such as quasi-linear preferences), the real price of capital is exactly constant.

\(^{18}\) There is an additional welfare difference between good and bad equilibria, related to the distribution of wealth across households. This effect contributes to increasing welfare in the bad equilibria. In a version of the model with features that rules out wealth effects, such as quasi-linear preferences, consumption misallocation remains the only source of welfare difference, and thus welfare is always lower in the bad equilibrium.

\(^{19}\) See Kubler and Schmedders (2010) for an introduction to Gröbner bases applied to the computation of equilibria in economic models.
4.3 Numerical example

Figure 2 and Table 1 show the results of a numerical simulated example of the model. I focus on two parameterizations that differ in the value of $k$ (i.e., the probability that household $h$ is hit by the preference shock $\bar{\epsilon} > 0$), $k = 0.5$ and $k = 0.85$.

I set $\alpha = 0.1$; therefore 10% of the banks are hit by $\bar{\psi}$. See Table 2 in Appendix E for the values of the other parameters.

For $k = 0.5$, the actual return on deposits of insolvent banks in the bad equilibrium is $r_t(\bar{\psi}) = -0.14 < 0$, and the other key endogenous variables are plotted in Figure 2. The economy is in steady-state in $t = 0$, experiences a crisis in $t = 1$, and then reverts to normal in $t = 2$. The top panel plots the prices $Q_t$, $p_t$, and the nominal return on capital $R^K_t$. The middle panel plots the evolution of capital held by banks (left panel) and households (right panel). During the crisis, banks have fewer resources because of the flight away from deposits. Therefore banks reduce holdings of capital with respect to pre-crisis level. Since the supply of capital is fixed, households must increase their holdings of capital in equilibrium. Interpreting the banking sector in the model as the shadow banking system in the US, and assuming that commercial banks are part of the household sector in the model, the result concerning capital holdings is consistent with the data analyzed by He et al. (2010).

The bottom left panel of Figure 2 plots some key variables in the money market: money held by households during the day, deposits, and a monetary aggregate denoted $M_1$. $M_1$ is defined as the sum of deposits and money held by households, in line with the standard definition of such a monetary aggregate.

The bad equilibrium is associated with a flight to liquidity. Households hold fewer deposits and more money in comparison to $t = 0$. As a result, the drop in $M_1$ with constant money supply $\bar{M}$ implies a drop in the money multiplier or, equivalently, a drop in velocity in the equation of exchange $\bar{M} \cdot velocity = p_t (\bar{K})$, which is consistent with empirical evidence about the Great Depression and the 2008-2009 US financial crisis.

The model qualitatively replicates some key facts of the money market during the Great Depression, plotted in the bottom right panel of Figure 2 (data are based on Friedman and Schwartz, 1970). During the Great Depression, the US economy experienced a drop in the total stock of money (top line) similar to the drop in $M_1$, a drop in total deposits (second line from the top), and an increase in currency held by the public (bottom line).

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20 He et al. (2010) find that, during the recent US financial crisis, securitized assets shifted from sectors dependent on repo financing to commercial banks.

21 In this context, the money multiplier is the ratio between $M_1$ and $\bar{M}$. 
Table 1 shows the result for $k = 0.85$. Under this parameterization, there exist two bad equilibria that I label “mild crisis” and “deep crisis.” The two bad equilibria are qualitatively identical, but in the deep crisis equilibrium, in comparison with the mild crisis, the drop in prices is more pronounced, the flight to liquidity and the drop in M1 are greater, and the return on deposits of insolvent banks is lower. The force that gives rise to multiple bad equilibria is analyzed in the next Section.

4.4 Understanding the multiplicity of bad equilibria

Proposition 4.2 suggests that the driving force behind the multiplicity of bad equilibria is a strategic complementarity across depositors. Recall that $H(b)$ denotes the depositors of bank $b$, and thus $\int_{H(b)} D_t^b db = D_t^b$ is the amount of deposits of bank $b$.

**Proposition 4.2.** Taking prices $Q_t$, $p_t$, and $R_t^K$ as given, the actual return on deposits $r_t^b$ of bank $b$ with negative net worth ($N_t^b < 0$) satisfies

$$\frac{\partial r_t^b}{\partial D_t^b} > 0.$$ 

The proof is provided in Appendix C. To understand the result, recall that an insolvent bank $b$ has preexisting losses that must be borne by depositors holding deposits at bank $b$. Taking prices as given (in particular, $Q_t$), Proposition 4.2 fixes
The economy experiences the crisis in period $t = 1$ and reverts to normal in $t+1$. Top panel: prices (price level $p_t$, nominal price of capital $Q_t$ and return on capital $R^D_t$). Middle panel: stock of capital held by banks (left) and households (right) at the end of the day market. Bottom left panel: money market (money supply $M$, deposits $\int M_D^b dh$, money held by households $\int M_H^b dh$, and $M1 = \text{deposits} + \text{money held by households}$). Parameter values: see Table 2, $k = 0.5$.

Bottom right panel: based on Table 2 from Friedman and Schwartz (1970) ("Money Stock" is the sum of currency held by the public and deposits); data are quarterly and seasonally adjusted, in billions of dollars.

The value of the net worth $N^b_t < 0$ (see equation (5)) which in turn represents the losses of the bank. If households decide to hold substantial deposits in bank $b$, each dollar of deposit bears a small loss; the opposite is also true. Consequently, the greater the deposits $\int M_D^b dh = D^b$ chosen by other depositors of bank $b$, the more willing household $h$ is to hold deposits issued by bank $b$, explaining the strategic complementarity. Such strategic complementarity does not arise in the good equilibrium because bank $b$ is solvent, and thus $r^b_t = R^D_t$ is independent of the choices of other depositors.

The result of Proposition 4.2 is a partial equilibrium exercise in the sense that it is derived fixing prices and analyzing the behavior of only one bank $b$ in the economy. Figure 3 shows how depositors affect
each other’s choices through a general equilibrium channel, computed as follows. First, fixing a value of deposits $\hat{D} \in \{0, D^t\}$, I force the representative household to hold deposits $\hat{D}$ (dropping the FOC with respect to $D^t_h$), and then I solve numerically for equilibrium prices $Q_t, p_t$, and the actual return $r_t$ ($\tilde{\psi}$). Second, I take as given the prices $Q_t, p_t$, and $r_t$ ($\tilde{\psi}$) just computed and allow a single household $h$ to take her optimal choices of money $M^t_h$, deposits $D^t_h$, and capital $K^t_h$ in a partial-equilibrium setting. Thus, I obtain a relation between $D^t_h$ and the value of $\tilde{\psi}$. A fixed point that satisfies $D^t_h = \hat{D}$ is an equilibrium of the model.

Figure 3 plots the choices of deposit $D^t_h$ by the representative household $h$ as a function of $\hat{D}$. Two bad equilibria arise if $k = 0.85$, whereas only one bad equilibrium arises if $k = 0.5$. To understand this difference, recall that the maximization problem of banks (6) is subject to the non-negativity constraint $K^t_b \geq 0$. I argue that this constraint imposes a lower bound on the value of deposits $\hat{D}$ for which an equilibrium with runs on a fraction of the banking system exists. To see this, combine the non-negativity constraint $K^t_b \geq 0$ with the budget constraint (7), the decision rule of banks $M^t_b = kD^t_b$ (from Proposition 3.1), the market clearing condition for deposits (23), and focus on a bank $b$ with negative net worth, $N^*_t = N_t(\tilde{\psi})$, obtaining:

$$\hat{D} \geq \frac{-N^*_t(\tilde{\psi})}{1 - \kappa} > 0.$$  \hspace{1cm} (27)

For the case $k = 0.5$, equation (27) is satisfied for $\hat{D} \geq 0.5$. Therefore it is not possible to fix a value of $\hat{D}$ lower than 0.5 to look for a deep crisis equilibrium. For $k = 0.85$, equation (27) is instead satisfied for $\hat{D} \geq 0.135$; the equilibrium value of deposits in the deep crisis equilibrium is 0.136, so it satisfies the constraint (27).

The value of $N_t(\tilde{\psi})$ is endogenous and thus (possibly) affected by monetary policy. In particular, monetary injections may reduce the absolute value of $N_t(\tilde{\psi})$, thus relaxing the constraint (27). For $k = 0.5$, some monetary injections give rise to a second bad equilibrium; see Sections 5.1.

When I solve the model with several values of the parameters, I can find at most two equilibria. It is not possible to rule out the existence of more than two bad equilibria using more general versions of the model, but understanding the multiplicity of bad equilibria in such models is beyond the focus of this paper.

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22 See Appendix D for a discussion of the bankless crisis equilibrium, where the non-negativity constraint on capital is binding.
5. Monetary policy

I consider the effects of a central bank that injects money into the economy during a crisis. The central bank announces a policy that will be implemented in the event of a panic, and can credibly commit to it. The central bank chooses money supply \( M_{t+j} = \tilde{M}(1 + \mu_{t+j}) \) for \( j = 0, 1, 2, \ldots \) where \( t \) is the period in which a crisis occurs. The full model with monetary injections is presented in Appendix F.

A monetary injection is characterized by two features. First, I distinguish between temporary and permanent increases in the money supply. A monetary injection is temporary if \( \mu_t > 0 \) and \( \mu_{t+j} = 0 \) (i.e., money reverts to the pre-crisis level when the panic ceases). A monetary injection is permanent if \( \mu_{t+j} = \mu \) for all \( j = 0, 1 \). Second, I distinguish between two methods to deliver the monetary injection: asset purchases and loans to banks.

In order to simplify the exposition, I further restrict Assumption 4.1 as follows.

**Assumption 5.1.** Assumption 4.1 holds, and equation (26) holds with equality.

A monetary injection creates inflationary pressure, counteracting the drop of nominal prices. Under Assumption 5.1, a sufficient condition to eliminate the bad equilibria is to inject enough money to achieve \( Q_t \geq Q^* \). Alternatively, it is possible to define a threshold for the price level (rather than for \( Q_t \)) such that targeting \( p_t \) above the threshold eliminates bad equilibria. A policy that achieves \( p_t \geq p^* \) also achieves \( Q_t \geq Q^* \), and it is thus sufficient to eliminate bad equilibria.

Note that relaxing Assumption 5.1 (i.e., allowing equation (26) to hold with strict inequality) let us to obtain similar results. A sufficient condition to eliminate bad equilibria is to achieve \( Q_t \) above a threshold, but in this case the threshold is lower than \( Q^* \) and depends on initial conditions. Intuitively, if equation (26) holds with strict inequality, banks have a larger net worth and thus more buffer to counteract the effects of the panic. Therefore a smaller monetary injection is sufficient to eliminate bad equilibria.

While it is clear, under Assumption 5.1, that a sufficiently large monetary injection achieves \( Q_t \geq Q^* \) and thus eliminates bad equilibria, the effects of a smaller monetary injection that results in \( Q_t < Q^* \) are less clear. Does it eliminate bad equilibria? If not, what are the effects on the endogenous variables?

A complete understanding of the positive implications of monetary injections requires analysis of these issues, independent of the optimality of policies. This analysis is also important if the central bank is unable or unwilling to achieve \( p_t \geq p^* \) for any considerations not captured by the model.

Sections 5.1 and 5.2 analyze monetary injections that result in \( Q_t < Q^* \) under Assumption 5.1 (similar results are obtained relaxing Assumption 5.1 and using the relevant threshold for \( Q_t \); see Appendix G.

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23 I ignore the values of \( \mu_{t+j} \) for \( j > 1 \) because they do not affect the results.

24 New Keynesian theory suggests that the optimal policy at zero nominal interest rates requires credibility to commit to future inflation (see, e.g., Krugman (1998) and Eggertsson and Woodford (2003)) and does not achieve the level of inflation that would prevail without the zero lower bound constraint. In practice, in the second half of 2008, the zero lower bound became binding, and the US experienced some deflation despite massive monetary injections by the Federal Reserve.
for more discussion). I focus on temporary monetary injections, much like the policies implemented during the recent US financial crisis. Section 5.1 analyzes temporary monetary injections implemented using asset purchases, and Section 5.2 analyzes temporary monetary injections implemented using loans to banks. Permanent monetary injections are discussed in Appendix H.

### 5.1 Asset purchases

The central bank buys capital in the market during the day of time $t$ and sells it during the day of $t+1$. Therefore money supply reverts to $M$ after the crisis. The returns from holding capital are rebated to households in $t+1$.

For each level of $\mu_t$ implemented with asset purchases and resulting in $Q_t < Q^*$, Figure 4 displays the outcome of the most important endogenous variables. More results are provided in Appendix G. When the central bank buys capital on the market, the demand for capital rises, and therefore the price $Q_t$ is higher than it would be in an economy without intervention. Due to the monetary injection, more money is in circulation, and thus the price level $p_t$ is also higher.

The higher $Q_t$ has two counteracting effects on deposits. First, the higher price of capital reduces losses of insolvent banks. Consequently, the actual return on deposits $r^{(-)}$ paid by insolvent banks is higher. This effect increases the demand for deposits from households since losses on deposits of insolvent banks are lower.

Second, using the fact that the monetary injection is temporary and thus $Q_{t+1} = Q^*$, the nominal return on capital, equation (4), becomes:

$$1 + R^K_t = \frac{Q^* + Zp_t}{Q_t}$$

The increase in $Q_t$ implied by the monetary injections reduces $R^K_t$ (recall from Section 4.2 that $p_t/Q_t$ is approximately constant, and thus the right-hand side of (28) is affected just by $Q^*/Q_t$). Since solvent banks pay the promised return on deposits $R^D_t = R^K_t$, the return $R^D_t$ declines with monetary injections as well. This effect reduces the demand for deposits by households, because $R^D_t$ is the return paid on deposits by solvent banks. The downward pressure on the demand for deposits is thus a consequence of the temporary nature of the monetary injection. In the numerical example that I

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25 In testimony before the Committee on Financial Services of the U.S. House of Representatives, Bernanke (2010) suggests that the monetary expansions of the Federal Reserve are temporary: “In due course [...] as the expansion matures the Federal Reserve will need to begin to tighten monetary conditions to prevent the development of inflationary pressures. The Federal Reserve has a number of tools that will enable it to firm the stance of policy at the appropriate time.”

26 I use here the parameters in Table 2 and $k = 0.5$. The results are robust to the choices of other parameter values with an important caveat. For some values of the parameters, a bad equilibrium does not exist for some $\mu$. See Appendix G for more discussion.
consider, the higher the monetary injection \( \mu_t \), the lower the deposits.\(^{27}\) (Amplification of the flight to liquidity arises also with loans to banks that do not eliminate bad equilibria; see Figure 5.)

The effectiveness of monetary policy is reduced by exacerbation of the flight to liquidity. Recall that a necessary condition for a bad equilibrium is \( r_t(\psi) < 0 \). On the one hand, monetary injections increase \( Q_t \), which in turn reduces the losses of insolvent banks, and thus it increases \( r_t(\psi) \). On the other hand, the reduction in deposits reduces \( r_t(\psi) \) (as a consequence of Proposition 4.2). In the numerical example, \( r_t(\psi) \) increases with \( \mu_t \), but the reduction in deposits partially offsets the positive effect of the higher \( Q_t \).

As discussed in Section 4.4, monetary injections affect the value of \( N_t(\psi) \), thus reducing the right-hand side of equation (27). Therefore, for sufficiently large monetary injections, asset purchases give rise to a deep crisis equilibrium for \( \kappa = 0.5 \) (see Figure 4), characterized by a larger flight to liquidity.

Can the central bank eliminate the bad equilibria using a temporary monetary injection implemented with asset purchases? Focusing on policies that result in \( Q_t < Q^* \), the simulations show that there exists at least one bad equilibrium.

![Figure 4: Effects of monetary policy - Asset purchases](image)

Horizontal axis: size of the monetary injection \( \mu_t \). Vertical axis: price of capital \( Q_t \) (left panel), deposit \( D^b_t \) (mid panel), and actual return on deposits of insolvent banks \( r_t(\psi) \) (right panel). The blue line represents the mild crisis equilibrium, the red line the deep crisis equilibrium, and the green dotted line the good equilibrium without monetary intervention. Parameter values: see Table 2, \( \kappa = 0.5 \).

5.2 Liquidity facility: loans to banks

The central bank provides loans to banks during the day at time \( t \). For each dollar borrowed at time \( t \), banks must repay \( 1 + R^b_t \) dollars during the day in \( t + 1 \). Banks can use funds borrowed from the central bank to hold money or buy capital. The budget constraint (7) of bank \( b \) during the day becomes:

\[\text{budget constraint:} \quad D^b_t + P^b_t \cdot Q_t + r_t(\psi) \cdot D^b_{t+1} = \]
If some banks are insolvent, I must consider the ability of the central bank to recover, in $t + 1$, the loans made in the day at time $t$. At one extreme, suppose loans from the central bank are senior with respect to depositors. The central bank is able to recover the full value of loans, and depositors split the value of assets after the central bank is repaid. Proposition 5.2 states that this case is equivalent to Section 5.1 in which the central bank buys capital on the market. The result follows from the fact that the central bank does not bear any losses of insolvent banks in both cases. The proof is provided in Appendix C.

**Proposition 5.2.** Given a policy $\mu_t > 0$ and $\mu_{t+j} = 0$ for all $j \geq 1$ implemented with asset purchases, if there exists a bad equilibrium, then the same equilibrium exists in an economy in which the same policy is implemented using loans to banks with higher seniority than deposits.

I now focus on the other extreme case: loans from the central bank have the same seniority as deposits. The central bank faces the actual return $r_t^{(L)} < 0$ on loans to insolvent banks. In this case, a moderate monetary injection eliminates bad equilibria even if it achieves $Q_t < Q^*$; see Figure 5. Thus, achieving $Q_t \geq Q^*$ is sufficient but not necessary to eliminate the bad equilibria using this policy.

When loans to banks have the same seniority as deposits, households are willing to hold more deposits (than under asset purchases) because some of the losses of the insolvent banks will be borne by the central bank. This behavior of the household sector has two effect. First, from a partial equilibrium perspective, the actual return on deposits of insolvent banks $r_t^{(L)}$ is higher and it turns positive for a moderate monetary injection (see right-hand panel of Figure 5, and see Proposition 4.2 for an explanation). Second, from a general equilibrium perspective, loans to banks break the strategic complementarity in the choice of deposits by the household sector, so only the mild crisis equilibrium exists.

In the model, loans to banks are more effective than asset purchases at eliminating the bad equilibria. This is the case even restricting attention to the mild crisis equilibrium; loans to banks eliminate the mild crisis equilibrium if $\mu_t > 0.3$, while asset purchases require much larger monetary injections ($\mu_t > 0.8$; see Figure 4). I leave to future research whether loans to banks is also a better policy in richer models, but I want to emphasize here an important case. In a model in which solvent banks are leveraged constrained, from a partial equilibrium perspective a bank might be unable to get a loan from the central bank without increasing leverage. Yet, loans to banks have two additional general equilibrium effects that contribute to reduce leverage. First, loans to banks are monetary injections that increase the equilibrium asset price $Q_t$, which in turn increases the net worth of banks. Second,

More precisely, if there exists two bad equilibria with a constant money supply $M_t = \bar{M}$, the strategic complementarity is weakened by a small monetary injection, but it disappears for a large enough $\mu_t$.

In this model, I refer to leverage as the ratio of assets to net worth at the end of the day market.
as I discussed above, monetary injections may reduce deposits and thus assets (because banks invest a fraction of deposits into capital). In the numerical example of Figure 5, the general equilibrium effects prevail, and thus loans to banks reduce leverage in equilibrium.
6. Conclusions

I have presented a new framework to analyze bank runs in a dynamic, general equilibrium model, and I have used it to study unconventional monetary policy during panic-based financial crises. In some circumstances, loans to banks with the same seniority as deposits eliminate the bad equilibria, while asset purchases do not. Moreover, for some parameter values, if a temporary monetary injection does not eliminate the bad equilibrium, it amplifies the flight to liquidity.

In order to provide loans to banks with the same seniority as deposits, the central bank must have the (legal) ability to take a loss on a loan to a particular bank. Factors that influence credibility become crucial. For instance, in a more general model in which some banks are insolvent even in the good equilibrium (because of fundamental shocks), must the central bank take losses on such fundamentally insolvent banks in order to credibly commit to loans with the same seniority as deposits? The story suggested by these questions appears consistent with what occurred in 2008.

The failure of Lehman Brothers in September 2008 might have communicated the inability of the Federal Reserve to make loans to banks facing risk of insolvency (contrary to what happened in March 2008 regarding Bear Stearns), thus opening up the possibility of a panic-based crisis.

This paper opens up three directions for future research. First, on the theoretical side, more work is required to identify the frictions that justify some of the assumptions that I have used, such as the nominal deposit contract. Second, on the empirical side, a richer version of the model would allow quantitative analysis in order to assess how panics, fundamental shocks, and other financial frictions contribute to financial crises. Third, on the policy side, the framework that I have presented can be used to analyze other policies such as capital requirements and equity injections.
References


Appendix

A  Extension: bankers and steady-state

The model described in the main text of the paper does not have a well-defined steady-state if banks have positive net worth, and it does not include a well-defined description of the ownership of banks. A minor extension to the model, presented in this Appendix, corrects for both shortcomings. This Appendix also characterize the steady-state and the good equilibrium prices and quantities as a function of the parameters.

The lack of a well-defined steady-state in the main part of the paper is related to the profits made by banks. Due to perfect competition in the banking market, banks do not make any extra profits. Nonetheless, if \( N^b_t > 0 \), then bank \( b \) owns some of the capital stock that it manages; since capital pays a return, bank \( b \) gets the return on capital, and its net worth grows over time. If \( R^D_t = R^K_t \) (which is the case in equilibrium), Proposition 3.1 implies that \( \mathbb{E}_\psi (N^b_{t+1}) = N^b_t (1 + R^K_t) \). To see this, rewrite equation (8) as:

\[
N^b_{t+1} = K^b_t (1 + \psi^b_{t+1}) Q_{t+1} + m^b_t - d^b_t \\
= K^b_t (1 + \psi^b_{t+1}) Q_{t+1} + Z K^b_t p_k (1 - \kappa) D^b_t (1 + R^D_t) \\
= (1 + \psi^b_{t+1}) \frac{N^b_t + (1 - \kappa) D^b_t}{Q_t} (Q_{t+1} + Z p_k) (1 - \kappa) D^b_t (1 + R^D_t)
\]

where the second line uses equation (9), and the third line uses Item 3 in Proposition 3.1. Taking expectations with respect to \( \psi^b_{t+1} \):

\[
\mathbb{E}_\psi (N^b_{t+1}) = \left[ N^b_t + (1 - \kappa) D^b_t \right] \frac{Q_{t+1} + Z p_k}{Q_t} - (1 - \kappa) D^b_t (1 + R^D_t)
\]

and the result follows by rearranging. Thus, abstracting from runs and bad equilibria, the net worth of banks increases over time.

Next, I introduce exit shocks that hit banks with probability \( \lambda \), so that some banks are subject to liquidation every periods, paying some dividends. It is then possible to impose restrictions on \( \lambda \) so that the overall net worth of the banking sector remains constant over time. The model of Section 2 is a special case of the extension below, obtained by setting \( \lambda = 0 \).

A.1  Environment

The economy is populated by households (double continuum, indexed by \( h \in \mathbb{H} = [0, 1] \times [0, 1] \)); banks; and a unit mass of bankers. During the day, bankers get dividends \( \pi_t \) from banks (described
Bankers are hand-to-mouth and use dividends $\pi_t$ to finance consumption $\frac{\pi_t}{p_t}$ at night.\footnote{The assumption of hand-to-mouth bankers simplifies the analysis and the exposition, but the results are unchanged in a model without bankers, where banks pay dividends to households and households trade claims on the stream of dividends of banks.}

Markets and interaction between households and banks works as described in Section 2.1. Households preferences are the same as in Section 2.1. The analysis of households is the same as in Section 3.2.

**Banks and exit shocks.** Between the night of $t - 1$ and the day of $t$, each bank is subject to an i.i.d. exit shock with probability $\lambda$ as represented in Figure 6. Assuming a law of large numbers, $\lambda$ is also the fraction of banks hit by the exit shock. Exiting banks are liquidated while the day market is open at time $t$, together with banks that were subject to runs in $t - 1$. Let $s \in S$ denote banks that are liquidated in $t$ (due to runs or exit shocks). Each surviving bank is instead “split” into $1/(1 - \lambda)$ new banks in order to keep constant the measure of surviving banks. Let $b \in B = [0, 1]$ denote active banks after this process.\footnote{If some bank is subject to runs in $t - 1$, so that the measure of $S$ is larger than $\lambda$ and the measure of banks after splitting is less than one, then new banks enter the market with no assets and no liabilities in order to keep the measure of active banks constant at one.}

After exit shocks are realized and surviving banks are split, shocks to capital $\psi_{t}^b$ and $\psi_{t}^s$ are realized.

**State variables of banks: law of motion.** Consider bank $b$ with capital, money, and deposits $K_t^b$, $m_t^b$, and $d_t^b$ at the end of the night. If bank $b$ is subject to a run in the night of time $t$, or subject to an exit shock at the end of time $t$, it becomes an exiting bank, denoted by $s \in S$, with state:

$$X_{t+1}^s = \{(K_t^b, m_t^b, d_t^b, \psi_{t+1}^s)\}$$

Below)
Otherwise, the bank is split into \(1/(1 - \lambda)\) new banks. The state variable of a bank \(b'\) that originates from the splitting of a bank \(b\) is:

\[
X_{t+1}^{b'} = \left\{ (1 - \lambda) \left( K^b_t, m^b_t, d^b_t \right), \psi_{t+1}^{b'} \right\}.
\]

(30)

**Liquidation and dividends.** Surviving banks operate as described in Section 2 of the text. Banks hit by the exit shock are liquidated when the day market is open at time \(t\). Liquidation works as follows. All assets of the bank are sold on the market, and deposits that were not withdrawn at night are repaid (if the value of assets is insufficient, depositors are repaid pro-rata). Any value left after repaying depositors contributes to the total dividends \(\pi_t\) that are paid to bankers.

Given the vector of state variables \(\{X^*_t\}_{s \in S}\) of banks under liquidation, the total value of dividends paid to bankers is:

\[
\pi_t = \int_S \max \{0, N^*_s\} \, ds,
\]

where net worth \(N^*_s\) is computed according to equation (5).

(31)

**State of the economy.** The aggregate state of the economy \(X_t\) at the beginning of the day is similar to the one defined in Section 2.4, but it also includes the state variables of banks under liquidation:

\[
X_t = \{ P^B_t, s_t, \{X^*_t\}_{s \in S} \}.
\]

**Restriction on parameters.** I impose restrictions on \(\lambda, \psi, \beta,\) and \(\kappa\) to ensure that there exists a well-defined steady-state.

**Assumption A.1.** \(\lambda = \frac{1-\beta}{\beta(1-\beta)(1/\kappa)}\).

**Assumption A.2.** The parameters \(\psi, \beta,\) and \(\kappa\) satisfy \(1 + \beta \kappa \psi + (1/\kappa) [\psi (1 - \beta) - \beta] > 0\).

A.2 Bank problem

Given net worth \(N^*_t\), the problem of bank \(b \in B\) is:

\[
\begin{align*}
\max_{D^b_t, M^b_t, K^b_t} \mathbb{E}_{\psi} \max & \left\{ 0, K^b_t \left( 1 + \psi_{t+1}^b \right) Q_{t+1} + m^b_t - d^b_t \right\} & \text{if } b \text{ will be liquidated in } t+1 \\
\max_{D^b_t, M^b_t, K^b_t} \mathbb{E}_{\psi} \left[ K^b_t \left( 1 + \psi_{t+1}^b \right) Q_{t+1} + m^b_t - d^b_t \right] & \text{if } b \text{ will not be liquidated in } t+1
\end{align*}
\]

(32)

\(\text{In the model of Section 2, which can be obtained setting } \lambda = 0, \text{ only banks subject to runs are liquidated. In equilibrium, banks subject to runs (if any) are insolvent, and thus no dividend is paid in that case.}\)
subject to (7), where $\mathbb{E}_\psi$ is the expectation taken with respect to $\psi_{t+1}^b$. If the bank will be liquidated in $t+1$, its objective is to maximize the value that will contribute to dividends, taking into account limited liability. Otherwise, its objective is to maximize the total net value of assets (assets minus deposits) before splitting. Lemma A.3 shows that the problem of banks can be formulated more simply.

Lemma A.3. If $r_t^b < R_t^D$ triggers the liquidation of bank $b$, then the problem of bank $b$ is equivalent to:

$$\max_{D_t^b, M_t^b, K_t^b} \mathbb{E}_\psi \{ 0, K_t^b (1 + \psi_{t+1}^b) Q_{t+1} + m_t^b - d_t^b \} \quad (33)$$

subject to (7).

Proof. If the bank is liquidated, then the result holds trivially. If the bank is not liquidated, then the solution to (33) is the same as the solution to (32) if (using $\psi_{t+1}^b = 0$ with probability one):

$$\left[ \max_{\{D_t^b, M_t^b, K_t^b\} \text{ s.t. (7)}} K_t^b Q_{t+1} + m_t^b - d_t^b \right] \geq 0$$

By contradiction, say that that is not the case. Then $K_t^b Q_{t+1} + m_t^b - d_t^b < 0$ or, rearranging and using (9), $K_t^b Q_{t+1} + m_t^b < (D_t^b - w_t^b) (1 + R_t^D)$. Thus, using equations (10) and (11), $r_t^b < R_t^D$, and thus the bank will be liquidated by assumption of the Lemma, which is a contradiction. $\square$

Note that the assumption Lemma A.3 is relevant, i.e., $r_t^b < R_t^D$ does trigger liquidation if households act optimally. First, if $r_t^b < 0$, the bank is subject to a run (Proposition 3.3) and thus liquidated. Second, if $0 \leq r_t^b < R_t^D$, depositors do not run at night; however, such a bank has negative net worth (because it does not have enough resources to repay the promised return $R_t^D$), and households will face losses on their deposits if, in the future, the bank is hit by an exit shock and liquidated (because the bank will not have enough resources to pay depositors). Anticipating this possibility, households withdraw all their deposit in the day of $t+1$, and by footnote 8 the bank is thus liquidated in $t+1$.

Finally, note that problem (33) is equivalent to problem (6) after replacing $N_{t+1}^b$ using equation (8).

A.3 Market clearing and equilibrium

Two of the market clearing conditions listed in Section 3.3 must be modified to account for dividends. In particular, equation (22) and (24) become, respectively:

$$\int_{\mathbb{H}} M_t^b db + \int_{\mathbb{H}} M_t^b dh = M_t - \pi_t \quad (34)$$
(because an amount \( \pi_t \) of money is used to pay dividends to bankers), and:

\[
\int_{\Pi} \frac{c_{ih}}{p_h} + \frac{\pi_t}{p_t} - ZK
\]

(because bankers are hand-to-mouth and their consumption is \( \pi_t/p_t \)).

The equilibrium definition is the same as Definition 3.5, with two modifications. First, the list of equilibrium objects includes also dividends \( \pi_t \). Second, equilibrium dividends must satisfy equation (31).

### A.4 Steady-state

First, I provide a definition of steady-state equilibrium (Definition A.4), and then I provide conditions on the state of the economy such that a steady-state equilibrium exists (Proposition A.5). Second, I characterize the unique steady-state equilibrium that satisfies Assumption 5.1 (Proposition A.6). Third, I describe the state of the economy when banks are hit by the idiosyncratic shocks \( \psi_t \) and \( \psi_t \), and I show that there exists a good equilibrium, which is also a steady-state (Proposition A.7). Finally, I describe conditions on the state of the economy such that prices and quantities are the same as in a steady-state, but the economy is in steady-state from \( t + 1 \) onward (Proposition A.8), which is relevant for the main analysis because this case encompasses what happens in \( t + 1 \) if a bad equilibrium is realized at time \( t \). Proofs are provided at the end.

**Definition A.4. (Steady-state equilibrium)** Given \( X_t \), a steady-state equilibrium is an equilibrium such that:

- prices are:
  \[
  Q_t = Q^* = \left[ \frac{\beta}{1-\beta} + \left( \frac{1}{\kappa} - 1 \right) \right] \frac{M}{K},
  \]
  \[
  p_t = p^* = \frac{M}{ZK},
  \]

- dividends paid by exiting banks are \( \pi_t - \pi^* (X_t) - \int_0^1 \left( K_t^e Q^* + d_t^{h-1} - m_t^{h-1} \right) ds \); bank \( b \in [0,1] \) has net worth \( N_t^b = K_t^b Q^* + m_t - d_t^{h-1} \geq 0 \) and choose \( D_t^b = D^* (X_t) = \frac{M - \pi^* (X_t)}{\kappa} \); and \( K_t^b = (N_t^b + (M - \pi^* (X_t)) (1/\kappa)) (1/Q^*) \) for all \( b \);

- household \( h \in H \) has beliefs \( \Pr_t^h (R_t, t^h = +\infty) = 1 \) and its choice is given by Proposition 3.3 and \( \eta_t^h = 0, \eta_t^L = (1-\beta)/(1-\beta(1-\kappa)), \eta_t^K = \beta\kappa/(1-\beta(1-\kappa)) \).
• actual return on deposits and limits on withdrawals are \( r_t^b = R^t \) for all \( b \) and \( l_t^h = +\infty \) for all \( h \);
• the state of the economy in \( t + 1 \) is \( X_{t+1} = X_t \).

Proposition A.5 provides conditions for the existence of a steady-state equilibrium.

**Proposition A.5.** (Existence of steady-state equilibrium) If Assumption A.1 holds and the state of the economy \( X_t \) satisfies:

1. \( \Pr^B_t \left( \{ X_t^b \mid i \beta_{t-1} < \frac{1}{\lambda} + (1 - \frac{1}{\lambda}) \frac{M}{K} + m_{t-1} - d_{t-1} \geq 0 \} \right) = 1; \)
2. \( m_{t-1} = K^b \left( M/K \right) \) for all \( b \in \mathbb{B}; \)
3. \( d_{t-1} - M (1 - \kappa) [K (1 - \beta - \kappa + 2 \beta \kappa) - (1 - \beta (1 - \kappa) \int \_b K_t^a db] / (K^b \kappa^2) > 0 \) for all \( b \in \mathbb{B}; \)
4. the distribution \( \Pr^B_t \) over the states of active banks is the same as the distribution over \( \{(1 - \lambda)X_t^b\}_{b \in \mathbb{B}} \) and the measure of banks in \( \mathbb{S} \) is \( \lambda \) (i.e., the set \( \mathbb{S} \) includes only banks hit by the exit shock);

then there exists a steady-state equilibrium.

Item 1 guarantees that all banks are solvent at the steady-state price \( q^* = \left[ \frac{1}{1 - \beta} + \frac{1}{\lambda} - 1 \right] \frac{M}{K} \).

Items 2 and 3 guarantee that the states \( \{ X_t^b \} \) of active banks are constant over time. Item 4 guarantees that the states of exiting banks \( \{ X_t^b \} \) are constant over time as well.

Next, I define the state of banks \( X^* \), and I show that if all banks have state \( X^* \) then a steady-state exists and Assumption 4.1 is satisfied. Moreover, no other steady-state satisfies Assumption 5.1. Let \( X^* = \{(K^*, m^*, d^*), 0\} \) where:

\[
K^* = \frac{K (1 - \beta) (1 - \kappa)}{1 - \beta (1 - \kappa (1 + \psi))}, \tag{36}
\]

\[
m^* = K^* \left( M/K \right), \tag{37}
\]

\[
d^* = \frac{M (1 - \kappa)}{\kappa (1 - \beta (1 - \kappa (1 + \psi)))}. \tag{38}
\]

**Proposition A.6.** (Main steady-state) If Assumptions A.1 and A.2 hold, and if the state of the economy satisfies \( \Pr^B_t (X_t^b - X^*) - 1, X_t^b - X^* \) for all \( b \in \mathbb{S} \), and the measure of banks in \( \mathbb{S} \) is \( \lambda \), there exists a unique steady-state equilibrium that satisfies Assumption 4.1.

The next Proposition guarantees the existence of a good equilibrium when the one-time unanticipated idiosyncratic shocks to capital \( \psi^a_t \in \{ \psi^a, \overline{\psi} \} \) are realized, and the economy is (in \( t - 1 \)) in the steady-state analyzed by Proposition A.6.
Proposition A.7. (Good equilibrium) If Assumptions A.1 and A.2 hold, and if the state of the economy satisfies:

\[ Pr_t^B \left( X_t^b = \left\{ (K^*, m^*, d^*) \right\}, \psi \right) = \alpha, \]
\[ Pr_t^B \left( X_t^b = \left\{ (K^*, m^*, d^*) \right\}, \overline{\psi} \right) = 1 - \alpha, \]
\[ X_t^e = \left\{ \left( \frac{K^*}{1 - \lambda}, \frac{m^*}{1 - \lambda}, \frac{d^*}{1 - \lambda} \right), \psi \right\} \text{ for the fraction } \alpha \text{ of banks under liquidation,} \]
\[ X_t^e = \left\{ \left( \frac{K^*}{1 - \lambda}, \frac{m^*}{1 - \lambda}, \frac{d^*}{1 - \lambda} \right), \overline{\psi} \right\} \text{ for the fraction } 1 - \alpha \text{ of banks under liquidation,} \]
and the measure of banks in \( S \) is \( \lambda \), there exists a steady-state equilibrium.

The last result of this Section encompasses the post-crisis case. After a bad equilibrium is realized, Proposition A.8 guarantees that the economy reaches a new steady-state.

Proposition A.8. (Equilibrium after a crisis) If Assumption A.1 holds and the state of the economy satisfies:

1. \( Pr_t^B \left( \left\{ X_t^b \right\} K_t^{b_S} \left( \frac{\rho}{1 - \beta} + \left( \frac{k}{\lambda} - 1 \right) \right) \overline{\Psi} + m_{t-1}^b - d_{t-1}^b \geq 0 \right) = 1; \)
2. for each \( b \in B \), \( d_{t-1}^b > 0 \) or \( \left( K_{t-1}^b, m_{t-1}^b, d_{t-1}^b \right) = (0, 0, 0); \)
3. \( \int_B \max \left\{ 0, \left( K_{t-1}^b Q^* + m_{t-1}^b - d_{t-1}^b \right) \right\} ds = \frac{\lambda}{1 - \lambda} \int_B \left( K_{t-1}^b Q^* + m_{t-1}^b - d_{t-1}^b \right) db; \)
then there exists an equilibrium with the same prices and quantities as the steady-state equilibrium but dividends that are defined as \( \pi_t = \int_B \max \left\{ 0, K_{t-1}^b Q^* + m_{t-1}^b - d_{t-1}^b \right\} ds \), and the economy is in a steady-state equilibrium from \( t + 1 \) onward.

The conditions of Proposition A.8 are satisfied in \( t + 1 \) if a crisis occurred at \( t \). Item 1 holds because only good banks survive to \( t + 1 \), while bad (insolvent) banks are subject to runs and liquidated. Item 2 holds because banks that originate from the splitting process have positive deposits, and new banks that enter the market to replace banks subjected to runs have zero assets and liabilities. Item 3 holds because, among the banks in \( S \) subject to liquidation, only banks hit by the exit shock have positive net worth (banks liquidated because of runs are insolvent), and their distribution is the same as active banks because the exit shocks are i.i.d.

Proofs.

Proof of Proposition A.5. Taking as given the return on deposits \( R_t^{D_S} = R_t^{F_S} = R^*, \) the choices of banks follow from Proposition 3.1. Item 1 guarantees that the net worth of banks is positive, using the definition of \( Q^* \). Plugging banks’ choices \( K_t^b \) and \( D_t^b \) (from Proposition 3.1) into equations (10) and (11), then \( r_t^b = R_t^{D_S} = R^* \) because \( N_t^b \geq 0. \)
Choices of households for money, deposits, and capital are proportional to wealth (Proposition 3.3) and denoted by \( M_t^h = \eta_t^M A_t^h, \) \( D_t^h = \eta_t^D A_t^h, \) and \( K_t^h = \eta_t^K A_t^h. \) Using equation (56) (from the proof of Proposition 3.3, see Appendix C) and households beliefs (that trivially satisfy rationality), the variables \( \eta_t^M, \eta_t^D, \) and \( \eta_t^K \) solve:

\[
\max_{\eta_t^M, \eta_t^D, \eta_t^K} \left\{ \kappa \varepsilon \log \left( \frac{\eta_t^M + \eta_t^D}{p_t} \right) + \kappa \frac{\beta}{1 - \beta} \log \left( \eta_t^K (1 + R_t^K) \right) + \right.
\]

\[
\left[ (1 - \kappa) \frac{\beta}{1 - \beta} \log \left( \eta_t^K (1 + R_t^K) + \eta_t^M + \eta_t^D (1 + R^*) \right) \right] \}
\]

subject to \( \eta_t^M + \eta_t^D + \eta_t^K = 1 \) and \( \eta_t^M, \eta_t^D, \eta_t^K \in [0,1]. \) First, I show that \( M_t^h = 0 \) or \( \eta_t^M = 0. \) Using the rearranged budget constraint \( A_t^h = 1 - \eta_t^M - \eta_t^D \) and imposing \( R_t^D = R_t^K = R^* , \) the first-order conditions with respect to \( \eta_t^M \) and \( \eta_t^D \) are:

\[
\text{FOC } \eta_t^M : \quad \frac{1}{\eta_t^D + \eta_t^M} \left( \frac{\beta R^* (1 - \kappa)}{(1 - \beta) (1 + R^K) (1 - \eta_t^M) + \eta_t^M} \right) - \frac{\beta \kappa}{(1 - \eta_t^D - \eta_t^M) (1 - \beta)} > 0
\]

(39)

\[
\text{FOC } \eta_t^D : \quad \frac{1}{\eta_t^D + \eta_t^M} - \frac{\beta \kappa}{(1 - \eta_t^D - \eta_t^M) (1 - \beta)} \]

(40)

Since \( \eta_t^D \) solves (40) equalized to zero, the FOC with respect to \( \eta_t^M \) is < 0. Thus it must be the case that \( \eta_t^M = 0 \) or \( M_t^h = 0 \) due to the non-negativity constraint on money. Moreover, using \( \eta_t^M = 0, \) equation (40) can be solved for \( \eta_t^D: \)

\[
\eta_t^D = \frac{1 - \beta}{1 - \beta (1 - \kappa)}
\]

(41)

and the value of \( \eta_t^K \) is computed using \( \eta_t^M + \eta_t^D + \eta_t^K = 1. \) Since all banks are solvent and pay the return \( R_t^h = R_t^D = R^* > 0 \) and \( \varepsilon_t^h = +\infty, \) the optimal withdrawal decision at night is to withdraw only if \( \varepsilon_t^h = \varepsilon \) (Proposition 3.3). As only impatient households withdraw, and banks hold \( M_t^b = \kappa D_t^b, \) then there are no runs and \( \varepsilon_t^b = +\infty. \)

The expression for dividends, equation (31), is trivially satisfied. It is useful, for future reference, to rewrite \( \pi_t \) as:

\[
\pi_t = \pi^* (X_t) = \int_{\mathbb{S}} (K_t^b Q^* + m_{t-1}^b - d_{t-1}^b) \, ds
\]

\[
= \frac{\lambda}{1 - \lambda} \int_{\mathbb{B}} (K_t^b Q^* + m_{t-1}^b - d_{t-1}^b) \, db
\]

(42)

where the last line follows from Item 4 and from the fact that the measure over \( \mathbb{B} = [0,1] \) is one,
while the measure over \( S \) is \( \lambda \).

Next, I check that market clearing conditions hold. Using the choices of banks in the definition of steady-state, money held by banks is:

\[
\int_{B} M_{t}^{b} \, db = \int_{B} \kappa D_{t}^{b} \, db - \frac{M - \pi^{\star} (X_{t})}{1 - \lambda}
\]

(43)

and \( \int_{B} M_{t}^{b} \, dh = 0 \). Thus money market clearing (34) holds.

To check the market clearing condition for deposits, I need to compute \( D_{t}^{b} \), which requires knowing the value of households' wealth. Using Corollary 3.4, I compute the wealth \( \overline{A}_{t} \) of the representative household summing the value of capital, money, and deposits of households. The wealth \( \overline{A}_{t} \) is:

\[
\overline{A}_{t} = \left( \frac{K^{b}}{1 - \lambda} \right) Q^{\star} + \left( \frac{M - \int_{B} \mu_{t-1}^{h} \, db}{1 - \lambda} \right) + \frac{\int_{B} \mu_{t-1}^{h} \, db}{1 - \lambda}.
\]

(44)

The first term on the right-hand side is the total supply of capital \( K \) minus the capital owned by active banks \( \int_{B} K^{b}_{t-1} \, db \) minus the capital owned by banks in liquidation \( \int_{B} K^{s}_{t} \, ds = \frac{1}{1 - \lambda} \int_{B} K^{b}_{t-1} \, db \) (which follows from Item 4); thus \( \overline{K} - \int_{B} K^{b}_{t-1} \, db - \frac{1}{1 - \lambda} \int_{B} K^{b}_{t-1} \, db = \frac{\int_{B} K^{b}_{t} \, db}{1 - \lambda} \). The second and third expressions on the right-hand side (money and deposits) are computed similarly to capital. Item 3 is sufficient to guarantee that households hold a strictly positive amount of deposits and capital (for capital, the result follows using the numerator in Item 3), and thus of money too (Item 2). To verify the market clearing condition for deposits (23), I need to check that:

\[
0 = \int_{B} D_{t}^{b} \, dh - \int_{B} D_{t}^{b} \, db - \eta^{D} \overline{A}_{t} - \frac{M - \pi^{\star} (X_{t})}{\kappa}
\]

(45)

where the last equality uses \( \int_{B} D_{t}^{b} \, dh = \eta^{D} \int_{B} A^{b}_{t} \, dh = \eta^{D} \overline{A}_{t} \) and the value of deposits of banks \( D_{t}^{b} \) in the definition of steady-state. Plugging (42) and (44) into (45) and rearranging, it is possible to verify that (45) holds.

In the goods market, total expenditure is the sum of money spent by households and bankers \( \int_{H} p_{t} c_{t}^{h} \, dh + \pi_{t} \):

\[
\int_{H} p_{t} c_{t}^{h} \, dh + \pi_{t} = \int_{H} w_{t}^{h} + \pi^{\star} (X_{t}) = \kappa \int_{B} D_{t}^{b} + \pi^{\star} (X_{t}) = \kappa \int_{B} D_{t}^{b} + \pi^{\star} (X_{t}) = \overline{M}
\]
where the first and second equalities use Proposition 3.3 and \( M^h = 0 \) for all \( h \); the third equality uses market clearing for deposits; and the last equality uses the previous result about the money market, equation (43). Therefore all the money supply \( \overline{M} \) is spent, and thus the total demand for consumption goods is \( \overline{M} = ZK \), where the equality uses the definition of \( p^* \). Since \( ZK \) is also the supply of consumption goods, market clearing holds in this market as well.

The market clearing for capital holds by Walras’ Law.

Finally, I show that the economy is in steady-state. Deposits at the end of the night, before the exit shocks, are:

\[
d_t^h = (D_t^h - u_t^h) (1 + R^*) \\
= (D_t^h - \kappa D_t^h) (1 + R^*) \\
= \frac{M - \pi^*(X_t)}{\kappa} (1 - \kappa) (1 + R^*) \\
= \frac{M - \frac{1 - \lambda}{\kappa} \int_k (K_{t-1}^h Q^* + m_{t-1}^h - d_{t-1}^h) db}{(1 - \kappa) (1 + R^*)} = \left( \frac{1}{1 - \lambda} \right) d_{t-1}^h
\]

where the second equality uses \( u_t^h = \int_{\Omega} u_t^h db = \kappa D_t^h \); the third equality uses \( D_t^h = \frac{M - \pi^*(X_t)}{\kappa} \) in the definition of steady-state; the fourth equality uses equation (42); and the last equality uses Assumption A.1 about \( \lambda \), Items 2 and 3, and rearranges. Money at the end of the night, before exit shocks, is:

\[
m_t^h = p^* ZK_t^h \\
= \frac{M}{ZK} Z \left( K_{t-1}^h Q^* + m_{t-1}^h - d_{t-1}^h + (1 - \kappa) \frac{M - \pi^*(X_t)}{\kappa} \right) Q^* \\
= \frac{M}{ZK} \frac{K_{t-1}^h \left[ \frac{\rho}{1 - \beta} + \left( \frac{1}{\alpha} - 1 \right) \frac{M}{ZK} + K_{t-1}^h \frac{M - \pi^*(X_t)}{\kappa} (1 - \kappa) + (1 - \kappa) \frac{M - \pi^*(X_t)}{\kappa} \right]}{\left[ \frac{\rho}{1 - \beta} + \left( \frac{1}{\alpha} - 1 \right) \frac{M}{ZK} \right]} \\
= \left( \frac{1}{1 - \lambda} \right) K_{t-1}^h \frac{M}{K}
\]

where the second equality uses the definition of \( p^* \) and the expression for \( K_t^h \) in the definition of steady-state; the third equality uses the definition of \( Q^* \) and Items 2 and 3; and the last equality rearranges using Assumption A.1 about \( \lambda \). Similarly, we can compute:

\[
K_t^h = \frac{K_{t-1}^h Q^* + m_{t-1}^h - d_{t-1}^h + (1 - \kappa) \frac{M - \pi^*(X_t)}{\kappa}}{Q^*} \\
= \left( \frac{1}{1 - \lambda} \right) K_{t-1}^h.
\]
Then, using the law of motion of the state vector of bank $b^i$ that originates from the splitting of bank $b$, equation (30):

$$X_{t+1}^{b^i} = \left\{ (1 - \lambda) \left( K_t^b, m_t^b, d_t^b, \psi_t^{b^i} \right), \psi_{t+1}^{b^i} \right\}$$

which is equal to $X_t^b$ (since $\psi_t^{b^i} = 0$ with probability one). Moreover, $\{X_{t+1}^s\}_{s \in S} = \{X_t^s\}_{s \in S}$ follows from Item 4, from the laws of motion of the states of banks, equations (29) and (30), and from the i.i.d. assumption of the exit shocks.

\[
\square
\]

**Proof of Proposition A.6.** The state of the economy satisfies all the conditions of Proposition A.5. In particular, Item 1 of Proposition A.5 follows from the definition of $X^s$: Item 2 of Proposition A.5 holds trivially; Item 3 of Proposition A.5 follows from the definition of $X^s$, and $d^s > 0$ due to Assumption A.2; Item 4 of Proposition A.5 holds trivially. Therefore, there exists a steady-state equilibrium. Moreover, $X^s$ satisfies Assumption 4.1. Uniqueness can be shown as follows. Combining equation (26) holding with equality, the fact that all banks must be alike (Assumption 4.1), and Items 2 and 3 from Proposition A.5, finding the state of the economy that gives rise to a steady-state equilibrium reduces to solving one linear equation, (26) holding with equality, in one unknown (the capital of bank $b$). Therefore the solution is unique.

\[
\square
\]

**Proof of Proposition A.7.** The conditions of Proposition A.5 are satisfied and can be checked as described in the Proof of Proposition A.6.

\[
\square
\]

**Proof of Proposition A.8.** The proof is very similar to the proof of Proposition A.5, with few differences. First, equation (42) is derived using Item 3 of Proposition A.8. Second, Item 2 of Proposition A.8 guarantees that $\overline{A}_t > 0$. Third, $X_{t+1}$ is not necessarily equal to $X_t$. As all the choices of banks at time $t$ are the same as in steady-state, then $K_t^b, m_t^b$, and $d_t^b$ are the same as in a steady-state equilibrium. Therefore $\{X_t^b\}_{b \in B}$ satisfies Items 1, 2, and 3 of Proposition A.5. $X_{t+1}$ satisfies also Item 4 of Proposition A.5 due to the i.i.d. assumption of the exit shocks.

\[
\square
\]

### B Discussion

This Appendix discusses assumptions related to the cash-in-advance constraint, the idiosyncratic shocks to capital, the demand-deposit contract, and the extension to multi-period crises.
B.1 Cash-in-advance constraint

The cash-in-advance constraint, combined with the unobservable preference shocks and the lack of market for capital at night, creates a precautionary demand for money. This is an important channel to give rise to multiple equilibria, although these assumptions could be replaced by others as long as a precautionary demand for money arises. Examples include transaction costs as in Alvarez et al. (2002); limits on the amount of assets that can be sold in each period as in Kiyotaki and Moore (2012); and search frictions that provide a proper micro-foundation for the role of money, as in Lagos and Wright (2005). Endogenizing which assets can be used for transactions (as in Lester et al., 2012) is another possible extension. Analysis of these approaches and of their consequences for policy is left for future research.

If other assets such as government bonds can be used for transactions (as suggested by the empirical evidence in Krishnamurthy and Vissing-Jorgensen, 2012), the model can be extended to replace money demand with a more general demand for assets that facilitate transactions.

B.2 Idiosyncratic shocks to capital $\psi_t^i$

The shock $\psi_t^i$ affects the quantity of capital of agent $i \in H \cup B$. After the shock is realized, agent $i$ holds a larger or a smaller quantity of capital. A similar result can be obtained if the shocks affect the quality (productivity) of capital. In a sense, having twice as many units of capital, for example, is equivalent to having the same amount of capital and doubling its productivity. Modeling heterogeneity in capital under asymmetric information, however, would be more complicated.

Alternative formulation. Realization of the shock $\psi_t^i$ is an agent-specific variable. However, an equivalent result arises when shocks are capital-specific rather than agent-specific, provided that there is correlation across shocks that hit each infinitesimal unit of capital held by agent $i$.

Consider the following formulation:

1. the state $X_t^i$ is defined as $X_t^i = \{(K_{t-1}^i, m_{t-1}^i, d_{t-1}^i), \Psi_t(K_{t-1}^i)\}$; that is, the realization of the shock $\Psi_t(K_{t-1}^i)$ is a function simply of the capital stock and does not depend directly on the identity of agent $i$;

2. there is a positive correlation across the shocks that hit each unit of capital held by agent $i$. Correlation across the shocks that hit each infinitesimal unit of capital held by agent $i$ is required to break the law of large numbers at the level of a single agent. A simple special case is $\Psi_t(K_{t-1}^i) = \psi_t K_{t-1}^i$ where $\psi_t^i$ is the shock that hits each unit of capital held by agent $i$. With this formulation, there is perfect correlation across the shocks that hit each unit of capital held by agent $i$. 

B.3 Demand-deposit contract

The model imposes the demand-deposit contract that banks offer to households, rather than deriving the contract from an explicit contracting problem. I leave for future research an exact characterization of the frictions that justify Assumptions 2.1, 2.3, and 2.4, but I nonetheless provide a brief discussion here. Assumption 2.2 is discussed in Section 2.1 of the body of the paper, and thus no further analysis is provided here.

Nominal demand-deposit contracts and sunspots. The assumption of nominal deposit contracts is crucial to the analysis because the multiplicity of equilibria relies on debt-deflation. If demand-deposit contracts can be made contingent on the state of the economy or on prices, then the bad equilibria do not exist. The banking literature has provided models such as Diamond and Rajan (2006) and Allen et al. (2013) in which a nominal deposit contract is equivalent to a state-contingent deposit contract and is strictly preferred if there are costs of writing state-contingent contracts. However, these costs are unmodeled in Diamond and Rajan (2006) and Allen et al. (2013), so the microfoundation of the strict optimality of the nominal deposit contract remains an open question.

Anyway, following Diamond and Rajan (2006) and Allen et al. (2013), it is possible to extend this paper to justify the nominal contract with a similar argument. Focusing just on the good equilibrium for simplicity, if the productivity of capital $Z$ is subject to aggregate shocks, the nominal contract achieves the ex-ante optimal allocation. Productivity shocks affect the price level $p_t$ through the market clearing condition for goods (the price level $p_t$ adjusts to obtain equality between consumption expenditure and the value of goods available), thus the real value of deposits $D_t^h/p_t$ adjusts and becomes de facto state-contingent. If it is costly to write state-contingent contracts, then the nominal contract is strictly preferred.

When also considering the possibility of a bad equilibrium, the optimal contract must trade off the gains of nominal contracts in normal times (avoiding costs associated with writing state-contingent contracts) with the costs associated with the bad equilibrium. If there is little probability of a bad equilibrium (more precisely, a low probability of a sunspot shock), then the ex-ante expected costs of the bad equilibrium are low. If the probability of a bad equilibrium is low and the cost of writing state-contingent contracts is high, then the nominal contract should arise as the optimal one. Looking at the empirical evidence in light of this theory, the fact that we observe nominal deposits in practice allows us to infer an upper bound on the probability of the sunspot shock. And this is consistent with the fact that systemic financial crises like the Great Depression or the Great Recession are rare events.

\[33\] A different approach to derive an upper bound on the probability of sunspots is discussed in Jovanovich and Tsyrennikov (2014).
Withdrawals and sequential service constraint. Diamond and Dybvig (1983) restrict the amount of withdrawals by impatient depositors to be independent of their position in line, thus limiting the analysis to suboptimal contracts. The bank run literature has analyzed conditions under which a run equilibrium exists if the optimal contract is used (see, e.g., Andolfatto et al., 2014; Green and Lin, 2003; Peck and Shell, 2003). Whether the run equilibrium exists or not in the Diamond and Dybvig (1983) model is not directly related to my analysis, because of the different channel that gives rise to runs and crises in my model.

Even if I remove both Assumption 2.3 and the sequential service constraint, a bad equilibrium may still arise. The incentive constraint required to truthfully reveal households’ type (patient/impatient) would imply \( w_t < D_t^b \) if the bank of household \( h \) is insolvent. This result would hold even if an insolvent bank decides to invest 100% of its assets in money during the day (see Footnote 12). Indeed, from a partial equilibrium perspective, an insolvent bank is “fundamentally insolvent,” in the sense that the value of its assets is lower than the value of its deposits. Therefore, households would face a withdrawal risk and thus a consumption risk, withdrawing \( w_t^b < D_t^b \) with probability \( \alpha \) (insolvent bank) or withdrawing \( w_t^h = D_t^h \) with probability \( 1 - \alpha \) (solvent bank). This outcome may still generate a flight to liquidity due to risk aversion, but the degree of risk aversion required to guarantee the existence of a bad equilibrium is a quantitative question.

A further extension is related to the dynamic nature of the model and to asymmetric information. If it takes more than one period for depositors to observe the shocks \( \psi_t^b \), then I conjecture that an insolvent bank that wants to survive as long as possible has an incentive to pay depositors fully (as long as it has money) rather than impose limits on their withdrawals, an outcome that resembles Assumption 2.3. If the bank does not do that and restricts withdrawals, de facto announcing its insolvency, it loses for sure the option value of surviving until \( t+1 \). Thus, the dynamic nature of the model may impose constraints on the optimal contract that arise from the behavior of banks, rather than just from the behavior of depositors as in Diamond and Dybvig (1983).

Return on deposits withdrawn. Assumption 2.4 is required mainly for tractability, and the channel that gives rise to multiple equilibria does not rely directly on it. Optimality in the model would require paying some returns also to depositors who withdraw, in order to compensate them for the preference shock that induces their consumption choices. Yet, it is possible to extend the model in a way that Assumption 2.4 is (weakly) optimal by gathering households in large families, such as in the model of Gertler and Kiyotaki (2013) where a continuum of households pool their wealth between the night of \( t-1 \) and the day of \( t \). In this setting, there is no need to compensate impatient households because the wealth is pooled anyway.
B.4 Multi-period crises

In the model, the (in)solvency of banks becomes common knowledge at night of time $t$. Only good banks survive to $t + 1$, and the economy reverts to normal in one period. Consider instead an extension to model a multi-period crisis. At night of time $t$, households learn the identity of only a fraction of insolvent banks (and run on them), while they obtain no information about all other banks in the economy. Therefore, at the beginning of $t + 1$, some banks in the economy are still insolvent, and the crisis lasts several periods.

Multi-period crises should not (qualitatively) affect the channel that gives rise to the multiplicity of equilibria. Therefore, I leave the analysis of multi-period crises and of their quantitative implications for future research.

C Proofs

Proof of Proposition 3.1. Since the shock $\psi_{t+1}^b = 0$ with probability one, I will ignore $\psi_{t+1}^b$ and, where relevant, equalities should be interpreted in the almost sure sense.

I first focus on the case $N_t^b \geq 0$ and I guess that $N_{t+1} \geq 0$; thus $\max \{0, N_{t+1}\} = N_{t+1}$. Therefore, the bank will be solvent in $t + 1$ and it will have enough resources to repay depositors because equations (10) and (11) imply that $r_t^b - R_t^P$. As a result, only impatient households (households with preference shock $\varepsilon_t^b = \hat{s}$) will withdraw at night, and by the law of large numbers assumed in Section 2.1, a fraction $\kappa$ of depositors are impatient. Therefore, bank $b$ chooses $M_t^b = \kappa D_t^b$ in order to have enough money to repay depositors. Withdrawals are given by $w_t^b = \kappa D_t^b$, and they satisfy the feasibility constraint $w_t^b \leq M_t^b$.

Using the definition of net worth $N_{t+1}^b$ based on equation (5) and the guess $N_{t+1}^b \geq 0$, the problem (6) of bank $b$ becomes:

$$\max_{D_t^b, M_t^b, R_t^b} K_t^b Q_{t+1} + m_t^b - d_t^b$$

subject to the non-negativity constraints $D_t^b \geq 0$, $M_t^b \geq 0$, $K_t^b \geq 0$, and the budget constraint (7). Using $M_t^b = w_t^b = \kappa D_t^b$, the definition of $R_t^K$ in equation (4), the budget constraint (7), and the definition of $m_t^b$ and $d_t^b$ in (9), the objective function becomes:

$$\max_{D_t^b} \left[ D_t^b (1 - \kappa) \left( R_t^K - R_t^P \right) + N_t^b \left( 1 + R_t^K \right) \right].$$

The objective function is linear in $D_t^b$, so the solution depends on the sign of $R_t^K - R_t^P$. If $R_t^K > R_t^P$, then $D_t^b = 0$. If $R_t^K < R_t^P$, then $D_t^b = +\infty$. If $R_t^K = R_t^P$, any amount $D_t^b \geq 0$ is a
solution. To verify the guess $N_{t+1}^b \geq 0$, note that if $R_t^K < R_t^D$ then $D_t^b - 0$, $M_t^b - \kappa D_t^b - 0$, and $N_{t+1}^b = N_t^b (1 + R_t^K) > 0$. If instead $R_t^K > R_t^D$, the bank issues $D_t^b = +\infty$; therefore $N_{t+1}^b = +\infty$. Finally, if $R_t^K = R_t^D$, then $N_{t+1}^b = N_t^b (1 + R_t^K) > 0$. Thus, the guess is verified.

If $N_t^b < 0$ and $R_t^D > R_t^K$, the bank can achieve $N_{t+1}^b = +\infty$ by issuing $D_t^b = +\infty$. Therefore guessing $N_{t+1}^b \geq 0$ and following the same steps as before allows us to verify the guess.

To complete the proof, I have to analyze the case $N_t^b < 0$ and $R_t^D \geq R_t^K$. To discuss this case, let me first state and prove an intermediate result.

**Lemma C.1.** If $N_t^b < 0$ and $R_t^D \geq R_t^K \geq 0$, the objective function of problem (6) achieves the value $\max \left\{0, N_{t+1}^b \right\} = 0$ for any possible choice $\{D_t^b, M_t^b, K_t^b\}$.

**Proof.** Using equation (8), the net worth at $t+1$ is:

$$
N_{t+1}^b = Q_t K_t^b \left( \frac{Q_{t+1} + Z_t y_t}{Q_t} \right) + (M_t^b - w_t^b) - (D_t^b - w_t^b) (1 + R_t^D)
= N_t^b (1 + R_t^K) + (D_t^b - M_t^b) (1 + R_t^K) + (M_t^b - w_t^b) - (D_t^b - w_t^b) (1 + R_t^D)
= N_t^b (1 + R_t^K) + R_t^K (D_t^b - M_t^b) - R_t^D (D_t^b - w_t^b))
$$

(46)

where the first equality uses equation (9), the production technology $y (K_t^b) = Z K_t^b$, and rearranges; the second equality uses the definition of $R_t^K$ in equation (4), the budget constraint (7), and the assumption of the Proposition that that the non-negativity constraint on capital does not bind; the third equality rearranges. Since feasibility requires $w_t^b \leq M_t^b, D_t^b - M_t^b \leq D_t^b - w_t^b$. Note that $D_t^b - M_t^b > 0$ using the budget constraint (7) and the assumption $N_t^b < 0$. Therefore, using the assumption of the Lemma 0 $\leq R_t^K \leq R_t^D$, we can write $R_t^K (D_t^b - M_t^b) - R_t^D (D_t^b - w_t^b) \leq 0$. Thus, using equation (46), $N_{t+1}^b < 0$. \(\square\)

Therefore, an insolvent bank (net worth $N_t^b < 0$) is indifferent among any choice, so it is (weakly) optimal to take the choices described by Proposition 3.1. \(\square\)

**Proof of Proposition 3.3.** I guess that:

- the value function has the form $V_t (A_t^b) = \tau^{-\eta} A_t^b + \Xi_t$ where $\Xi_t$ is independent of $A_t^b$;
- the policy functions are $M_t^b = \eta_t^M A_t^b$, $D_t^b = \eta_t^D A_t^b$ and $K_t^b = \frac{\sigma_t^A A_t^b}{\eta_t} = 1$ where $\eta_t^M, \eta_t^D$, and $\eta_t$ are independent of $A_t^b$ and $\eta_t^M + \eta_t^D + \eta_t = 1$.

To verify the guess and prove the other results of the Proposition, I proceed backward. I analyze first the problem at night, i.e., choices of withdrawals $w_t^b (n_t^b)$ and consumption $c_t^b (n_t^b)$.

If $c_t^b - \tilde{z}$, I can rewrite the right-hand side of the Bellman equation (14) and omit terms that do not depend on choices taken at night when $c_t^b = \tilde{z}$. I use the notation $n_t^b | \tilde{z}$ to denote a state of
the world \( n_{t}^{h} \in \mathcal{N} \) for household \( h \), conditional on \( \varepsilon_{t}^{h} = \varepsilon \). Using the assumption that the shock \( \psi_{t+1}^{h} = 0 \) with probability one, equation (18) and the guess about the value function:

\[
\max_{w_{h}(n_{t}^{h}|\varepsilon), c_{h}(n_{t}^{h}|\varepsilon)} \left[ \varepsilon \log c^{h}(n_{t}^{h}|\varepsilon) + \frac{\beta}{1 - \beta} \log \left( K_{t+1}^{h}Q_{t+1} + d^{h}(n_{t}^{h}|\varepsilon) + m^{h}(n_{t}^{h}|\varepsilon) \right) \right] \tag{47}
\]

subject to the cash-in-advance constraint and the constraint on withdrawals:

\[
p_{t}c^{h}(n_{t}^{h}|\varepsilon) \leq M_{t}^{h} + w^{h}(n_{t}^{h}|\varepsilon) \tag{48}
\]

\[
0 \leq w^{h}(n_{t}^{h}|\varepsilon) \leq \min \{ D_{t}^{h}, l_{t}^{h} \} \tag{49}
\]

where, similarly to (19) and (20):

\[
c^{h}(n_{t}^{h}|\varepsilon) = [D_{t}^{h} - w^{h}(n_{t}^{h}|\varepsilon)] \left( 1 + \frac{b(h)}{r_{t}^{b(h)}} \right) \tag{50}
\]

\[
m^{h}(n_{t}^{h}|\varepsilon) = [M_{t}^{h} + w^{h}(n_{t}^{h}|\varepsilon)] \left( 1 + \frac{b(h)}{r_{t}^{b(h)}} \right) \tag{51}
\]

I argue that the cash-in-advance constraint (48) must be satisfied with equality; otherwise the household would be better off by changing its decisions during the day. If there is unspent cash when \( \varepsilon_{t}^{h} = \varepsilon \) the household could reduce its holdings of money \( M_{t}^{h} \) and its withdrawals (thus, reduce deposits \( D_{t}^{h} \)) and invest more in capital. Therefore equations (48) and (51) become:

\[
c^{h}(n_{t}^{h}|\varepsilon) = \frac{M_{t}^{h} + w^{h}(n_{t}^{h}|\varepsilon)}{p_{t}} \tag{52}
\]

and

\[
m^{h}(n_{t}^{h}|\varepsilon) = p_{t}(ZK_{t}^{h}) \tag{53}
\]

Plugging (50), (52), and (53) into (47), using the guesses about \( M_{t}^{h}, D_{t}^{h}, \) and \( K_{t}^{h} \), and using the definition of \( R_{t}^{h} \) in equation (4):

\[
\max_{w_{h}(n_{t}^{h}|\varepsilon)} \left[ \varepsilon \log \left( \frac{\eta_{t}^{M}A_{t}^{h} + w^{h}(n_{t}^{h}|\varepsilon)}{p_{t}} \right) \right.

\left. + \frac{\beta}{1 - \beta} \log \left( \eta_{t}^{K}A_{t}^{h}(1 + R_{t}^{h}) + [\eta_{t}^{D}A_{t}^{h} - w^{h}(n_{t}^{h}|\varepsilon)] \left( 1 + r_{t}^{b(h)} \right) \right) \right] \tag{54}
\]

subject to the constraint on withdrawals (49).

I claim that, if \( l_{t}^{h} = +\infty \), then \( w^{h}(n_{t}^{h}|\varepsilon), l_{t}^{h} = +\infty \) = \( D_{t}^{h} = \eta_{t}^{D}A_{t}^{h} \). Since the actual return on deposits \( r_{t}^{b(h)} \) is weakly lower than the promised return \( R_{t}^{D} \), i.e., \( r_{t}^{b(h)} \leq R_{t}^{D} \), and since the
Proposition considers the case $R_t^D - R_t^K$, then $r_t^{(h)} \leq R_t^K$. That is, the return on deposits is always weakly lower than the return on capital. Using also Assumption 3.2, households decide to buy deposits only to finance consumption expenditures in case their realized preference shock is $\varepsilon_t^h = \varepsilon$. Therefore, for the case $\varepsilon_t^h = \varepsilon$ and $l_t^h = +\infty$, the entire amount of deposits $D_t^h$ is withdrawn and used for deposits.

If $l_t^h = 0$, the bank has no cash when household $h$ is served, so withdrawals are constrained to be zero, $w_t^h (n_t^h|\varepsilon, l_t^h - 0) - 0$.

If the preference shock is zero, $\varepsilon_t^h = \varepsilon = 0$, then $c_t^h (n_t^h|\varepsilon) = 0$ since the household gets no utility from consumption. The right-hand side of the Bellman equation (14) can be written, similarly to equation (54):

$$\max_{w_t^h(n_t^h|\varepsilon)} \log \left( n_t^h A_t^h \left( 1 + P_t^K \right) + n_t^M A_t^h + u_t^h (n_t^h|\varepsilon) + \left[ n_t^D A_t^h - w_t^h (n_t^h|\varepsilon) \right] \left( 1 + r_t^{(h)} \right) \right)$$

subject to $0 \leq w_t^h (n_t^h|\varepsilon) \leq \min \{ D_t^h, l_t^h \}$. Taking a monotonic transformation of the objective function (to get rid of the log) and omitting terms that do not depend on $w_t^h (n_t^h|\varepsilon)$:

$$\max_{w_t^h(n_t^h|\varepsilon)} -w_t^h (n_t^h|\varepsilon) \cdot r_t^{(h)} \quad \text{s.t.} \quad 0 \leq w_t^h (n_t^h|\varepsilon) \leq \min \{ D_t^h, l_t^h \}.$$

Because of the linearity, the solution is always at the corner, i.e., either zero or $\min \{ D_t^h, l_t^h \}$:

$$w_t^h (n_t^h|\varepsilon) = \begin{cases} D_t^h = n_t^D A_t^h & \text{if } r_t^{(h)} < 0 \text{ and } l_t^h = +\infty \\ 0 & \text{if } r_t^{(h)} < 0 \text{ and } l_t^h = 0 \\ 0 & \text{if } r_t^{(h)} > 0 \text{ and } l_t^h \in \{ 0, +\infty \}. \end{cases}$$

Note that if $r_t^{(h)} < 0$, the return on deposits not withdrawn is negative and thus lower than the return on money (the return on money is zero). If $r_t^{(h)} < 0$, household $h$ runs to withdraw as much money as possible, provided that the household is able to reach its own bank while the bank still has money to pay withdrawals.

To sum up, choices at night can be classified into four cases.
1. $\varepsilon_t^h = \varepsilon, r_t^{(h)} \in \mathbb{R}$ and $l_t^h = +\infty$. Withdrawals are $w_t^h = D_t^h$, and consumption is $c_t^h = \frac{M_t^h + D_t^h}{p_t}$;
2. $\varepsilon_t^h = \varepsilon, r_t^{(h)} \in \mathbb{R}$ and $l_t^h = 0$. Withdrawals are $w_t^h = 0$, and consumption is $c_t^h = \frac{M_t^h}{p_t}$;
3. $\varepsilon_t^h = \varepsilon, r_t^{(h)} < 0$ and $l_t^h = +\infty$. Withdrawals are $w_t^h = D_t^h$, and consumption is $c_t^h = 0$ (run);
4. $\varepsilon_t^h = \varepsilon = 0, (r_t^{(h)}, l_t^h)$ such that $r_t^{(h)} \geq 0$ and/or $l_t^h = 0$. Withdrawals are $w_t^h = 0$, and
consumption is $x_t^h = 0$.

Next, I take as given the choices at night and I analyze the decisions during the day, in order to verify the guesses about $M_t^h$, $D_t^h$, and $K_t^h$, and about the value function. I use the distinctions among the four cases at night, taking into account the beliefs of households $\Pr_t^h$ about $x_{i_t}^{l(k)}$ and $x_{i_t}^{h}$, and the probability distribution over $e_t^h$ given by (1). The maximization problem during the day is:

$$
\max_{\eta_t^M, \eta_t^D, \eta_t^K} \left\{ \Pr_t^h \left( x_{i_t}^{h(k)} \in \mathbb{R}, x_{i_t}^{h} = +\infty \right) \kappa \left[ \varepsilon \log \left( \frac{\eta_t^M A_t^h + \eta_t^D A_t^h}{p_t} \right) + \frac{\beta}{1-\beta} \log \left( \eta_t^K A_t^h \left( 1 + R_t^K \right) \right) \right] + \\
\Pr_t^h \left( x_{i_t}^{l(k)} \in \mathbb{R}, x_{i_t}^{h} = 0 \right) \kappa \left[ \varepsilon \log \left( \frac{\eta_t^M A_t^h}{p_t} \right) + \frac{\beta}{1-\beta} \log \left( \eta_t^K A_t^h \left( 1 + R_t^K \right) + \eta_t^D A_t^h \left( 1 + x_{i_t}^{l(b(h))} \right) \right) \right] + \\
\Pr_t^h \left( x_{i_t}^{l(k)} < 0, x_{i_t}^{h} = +\infty \right) (1-\kappa) \left[ \frac{\beta}{1-\beta} \log \left( \eta_t^K A_t^h \left( 1 + R_t^K \right) + \eta_t^M A_t^h + \eta_t^D A_t^h \left( 1 + x_{i_t}^{l(b(h))} \right) \right) \right] + \\
\left[ 1 - \Pr_t^h \left( x_{i_t}^{l(k)} < 0, x_{i_t}^{h} = +\infty \right) \right] (1-\kappa) \left[ \frac{\beta}{1-\beta} \log \left( \eta_t^K A_t^h \left( 1 + R_t^K \right) + \eta_t^M A_t^h + \eta_t^D A_t^h \left( 1 + x_{i_t}^{l(b(h))} \right) \right) \right] \right\} 
$$

(55)

subject to the budget constraint (15) which can be written:

$$\eta_t^M A_t^h + \eta_t^D A_t^h + \eta_t^K A_t^h \leq A_t^h.$$ 

All terms $A_t^h$ can be factored out of the logs in the objective function, implying:

$$
\max_{\eta_t^M, \eta_t^D, \eta_t^K} \left\{ \Pr_t^h \left( x_{i_t}^{h(k)} \in \mathbb{R}, x_{i_t}^{h} = +\infty \right) \kappa \left[ \varepsilon \log \left( \frac{\eta_t^M + \eta_t^D}{p_t} \right) + \frac{\beta}{1-\beta} \log \left( \eta_t^K \right) \right] + \\
\Pr_t^h \left( x_{i_t}^{l(k)} \in \mathbb{R}, x_{i_t}^{h} = 0 \right) \kappa \left[ \varepsilon \log \left( \frac{\eta_t^M}{p_t} \right) + \frac{\beta}{1-\beta} \log \left( \eta_t^K \left( 1 + R_t^K \right) + \eta_t^D \left( 1 + x_{i_t}^{l(b(h))} \right) \right) \right] + \\
\Pr_t^h \left( x_{i_t}^{l(k)} < 0, x_{i_t}^{h} = +\infty \right) (1-\kappa) \left[ \frac{\beta}{1-\beta} \log \left( \eta_t^K \left( 1 + R_t^K \right) + \eta_t^M + \eta_t^D \left( 1 + x_{i_t}^{l(b(h))} \right) \right) \right] + \\
\left[ 1 - \Pr_t^h \left( x_{i_t}^{l(k)} < 0, x_{i_t}^{h} = +\infty \right) \right] (1-\kappa) \left[ \frac{\beta}{1-\beta} \log \left( \eta_t^K \left( 1 + R_t^K \right) + \eta_t^M + \eta_t^D \left( 1 + x_{i_t}^{l(b(h))} \right) \right) \right] \right\} 
$$

(56)

and the budget constraint can be written (with equality) $\eta_t^M + \eta_t^D + \eta_t^K = 1$. The objective function and the budget constraint are independent of $A_t^h$, and thus the optimal $\eta_t^M$, $\eta_t^D$, and $\eta_t^K$ are independent of $A_t^h$ too. The variables $\eta_t^M$, $\eta_t^D$, and $\eta_t^K \in [0,1]$ to satisfy the non-negativity constraints on money, deposits, and capital. The guesses about the policy functions are thus verified. The optimal values of $\eta_t^M$, $\eta_t^D$, and $\eta_t^K$ can be computed by taking first-order conditions of (56) and solving the resulting system of equations, subject to $\eta_t^M + \eta_t^D + \eta_t^K = 1$ and $\eta_t^M$, $\eta_t^D$, and $\eta_t^K \in [0,1]$.

Finally, I verify the guess for the value function. To do so, I rewrite the Bellman equation (14)
omitting terms that do not depend on \( A^b_t \). Using equation (55) and the guess for \( V_t \ (A^b_t) \):

\[
\frac{1}{1 - \beta} \log A^b_t = \kappa \varepsilon \log (A^b_t) + \frac{\beta}{1 - \beta} \log A_t^b
\]

this expression holds true because \( \kappa \varepsilon - 1 \) from equation (2), confirming the guess about the value function. \( \square \)

**Proof of Proposition 4.2.** The variable \( \tau^b_t \) can be written:

\[
1 + \tau^b_t = \frac{K^b_t (Q_{t+1} + Z_{Pt})}{D^b_t - u^b_t} = \\
= \left( \frac{Q_{t+1} + Z_{Pt}}{Q_t} \right) \left( \frac{D^b_t + (1 - \kappa) D^b_t}{D^b_t (1 - \kappa)} \right) = \left( \frac{Q^* + Z_{Pt}}{Q_t} \right) \left( 1 + \frac{N^b_t}{D^b_t (1 - \kappa)} \right) \quad (57)
\]

where the first equality uses the definition of \(\tau^b_t\) in equation (11); the second equality uses the results \( K^b_t = (N^b_t + (1 - \kappa) D^b_t) / Q_t \) and \( u^b_t = M^b_t = \kappa D^b_t \) of Proposition 3.1; and the third equality rearranges and notes that the term in the first parentheses is the nominal return on capital, using equation (4).

Since prices are fixed by assumption (and thus \( N^b_t < 0 \) is fixed as well because it depends on state variables and on \( Q_t \); see equation (5)) and \( D^b_t \geq 0 \) due to a non-negativity constraint,

\[
\frac{\partial \tau^b_t}{\partial \left[ \int_{H(b)} D^b_t dh_t \right]} = \frac{\partial \tau^b_t}{\partial D^b_t} = (1 + R^b_t) \left( -\frac{N^b_t}{[D^b_t (1 - \kappa)]^2} \right) > 0.
\]

\( \square \)

**Proof of Proposition 5.2.** By assumption, there exists an equilibrium with money supply \( M_t = \bar{M} (1 + \mu_t) \), in which the monetary injection \( \mu_t \bar{M} \) is implemented with asset purchases. The 16 equations that characterize the equilibrium are described in Appendix E.7. Among them, there is:

1. the budget constraint of banks, holding with equality:

\[
M^b_t + Q_t K^b_t = D^b_t + N^b_t;
\]

2. the actual return on deposits paid by insolvent banks:

\[
1 + r^b_t = \frac{K^b_t (Q^* + Z_{Pt})}{D^b_t - u^b_t}.
\]
Let $L^b_t = L^b_{eq} - \mu_i M^i$ and define $\tilde{K}^b_t = \frac{K^b_t}{Q_t}$. Then I can add $L^b_t$ on both sides of equation (58) and, using the definition of $\tilde{K}^b_t$:

$$M^b_t + Q_t R^b_t + \tilde{K}^b_t = D^b_t + L^b_t + N^b_t.$$  

Defining $\tilde{K}^b_t = K^b_t + \tilde{K}^b_t$:

$$M^b_t + Q_t K^b_t = D^b_t + L^b_t + N^b_t.$$  

(60)

From Section F.2, given prices $R^D_t = R^CB_t = R^K_t$, bank $b$ chooses $M^b_t = \kappa D^b_t$; any $D^b_t$ is a solution; any $L^b_t$ is a solution; and $\tilde{K}^b_t$ solves equation (60). Therefore, the choices $D^b_t$ and $M^b_t$ from the equilibrium with asset purchases, together with $K^b_t = K^b_t + \tilde{K}^b_t$ and $L^b_t = L^b_{eq}$, are optimal for banks.

Next, adding and subtracting $L^b_t (1 + R^K_t)$ on the numerator on the right-hand side of equation (59), and using the definition of $K^b_t$:

$$1 + r^b_t = \frac{K^b_t (Q^* + Z_{ps}) + \tilde{K}^b_t Q_t (1 + R^K_t) - L^b_t (1 + R^K_t)}{D^b_t - w^b_t} - \frac{K^b_t (Q^* + Z_{ps}) + \tilde{K}^b_t Q_t (\frac{Q^* + Z_{ps}}{Q_t}) - L^b_t (1 + R^K_t)}{D^b_t - w^b_t}$$

$$= \frac{K^b_t (Q^* + Z_{ps}) - L^b_t (1 + R^K_t)}{D^b_t - w^b_t}$$  

(61)

where the second line uses equation (28) and the last line uses $\tilde{K}^b_t = K^b_t + \tilde{K}^b_t$. Comparing the last result with equation (78), $r^b_t$ is also the actual return on deposits in an economy where banks get loans $L^b_t$ that are senior to deposits.

Equations (60) and (61), together with the remaining 14 equations that described the original equilibrium, describe an equilibrium with loans to banks with the same prices and quantities but the amount of capital chosen by banks ($K^b_t$ is replaced by $\tilde{K}^b_t$). The choices of the representative household are unchanged because they depend only on prices and on the household’s own state variable $X^h_t$. The market clearing conditions for money, deposits, and consumption goods are also unchanged. The market clearing for capital (83) in the equilibrium with loans to banks must hold by Walras’ Law.

**D Bankless crisis equilibrium**

This Section presents a bad equilibrium in which all banks are insolvent during the day of time $t$. The banking system is shut down for a period, and the economy reverts to normal in $t + 1$. Since
all banks are insolvent, the existence of this equilibrium does not require the shocks to capital $\Psi$ and $\overline{\Psi}$ to hit the economy, and it does not hinge on asymmetric information as well. The precise conditions for existence are stated in Proposition D.1 below. In order to clarify how these results arise, I first discuss in more detail the liquidation of insolvent banks (as described by footnote 8) and the role of the non-negativity constraints on money and capital in the problem of bank (6).

Consider an active bank $b$ with initial deposits $d_{t-1}^b > 0$ and negative net worth, $N_t^b < 0$. The bank is able to operate if $D_t^b$ is large enough and $N_t^b + D_t^b > 0$. In this case, the bank has resources to hold positive amounts of money and capital. If, on the contrary, the value of deposits $D_t^b$ is small and $N_t^b + D_t^b < 0$, the non-negativity constraints on money and capital $M_t^b \geq 0$ and $K_t^b \geq 0$ become binding. Recall that $d_{t-1}^b$ represent the pre-existing deposits at bank $b$; if any pre-existing deposits are rolled over during the day by depositors, the bank might not have enough resources to pay depositors. Footnote 8 states that, if this occurs, bank $b$ is shut down in the day of time $t$ (in the sense that it is forced to choose $M_t^b = 0$ and $K_t^b = 0$), and it is liquidated immediately (in the sense that preexisting deposits are repaid pro-rata using the value of the current assets of bank $b$, defined by $K_{t-1}^b (1 + \psi_t^b) Q_t + m_{t-1}^b$). If household $h$ chooses deposits $D_t^b > 0$ at a bank $b(h)$ that is liquidated in the day of time $t$, then the household is depositing part of its wealth at a bank in which $M_t^{b(h)} = 0$ and $K_t^{b(h)} = 0$ and in which all the available resources are used to pay preexisting depositors. Thus, all the deposits $D_t^b$ are lost (implying $q_t^{b(h)} = -1$) and cannot be withdrawn at night because the bank is forced to hold $M_t^{b(h)} = 0$; thus $d_t^b = 0$.34

Proposition D.1 states the condition for the existence of the bankless crisis equilibrium and describes it.

**Proposition D.1. (Bankless equilibrium) If the state of the economy $X_t^b$ satisfies:**

\[
pr_t^b \left( X_t^b \left| K_{t-1}^b (1 + \psi_t^b) \left[ 1 - \frac{\kappa (1 - \kappa)}{1 - \beta + 2\beta \kappa (1 - \kappa) + \kappa^2} \right] \frac{M_t}{K_t} \frac{\beta}{1 - \beta} + m_t^b - d_{t-1}^b < 0 \right. \right) = 1
\]

and:

\[
K_t^b \left[ 1 - \frac{\kappa (1 - \kappa)}{1 - \beta + 2\beta \kappa (1 - \kappa) + \kappa^2} \right] \frac{M_t}{K_t} \frac{\beta}{1 - \beta} + m_t^b - d_t^b < 0
\]

for all $b \in S$, then there exists an equilibrium such that:

- prices are:

\[
Q_t = M_t \frac{\beta}{K_t} \frac{1 + \frac{\kappa (1 - \kappa)}{1 - \beta + 2\beta \kappa (1 - \kappa) + \kappa^2}}{1 - \beta - 2\beta \kappa (1 - \kappa) + \kappa^2}
\]

34 Alternatively, the results $q_t^{b(h)} = -1$ can be obtained as follows. Consider a bank $b$ with $N_t^b < 0$, and assume that bank $b$ issues $D_t^b = \delta$, where $\delta > 0$ but small, and then take the limit as $\delta \to 0$. Since $\delta$ is small, the non-negativity constraint on money and capital is still binding, so $M_t^b = 0$ and $K_t^b = 0$. Using equation (11), then $q_t^b \to -1$ as $\delta \to 0$. 
\[ P_t = \frac{\kappa M}{Z K}, \]
\[ 1 + R_t^D = 1 + R_t^K = \frac{Q^* + Z\pi_t}{Q_t}, \]

where \( Q^* \) is the steady-state price (see Definition A.4).

- dividends paid by exiting banks are \( \pi_t = 0 \);
- all banks subject to exit shocks have negative net worth, \( N_t^s < 0 \) for all \( s \in S \);
- all active banks have negative net worth, \( N_t^b < 0 \) for all \( b \in B \) and choose \( M_t^b = D_t^b = K_t^b - 0 \);
- household \( h \in H \) has beliefs \( P_{i_t}^h (r_t^h = -1, l_t^h = 0) = 1 \); and its choice is given by Proposition 3.3, \( \eta_t^D = 0 \), \( \eta_t^K = 1 - \eta_t^M \), and:

\[
\eta_t^M = \frac{(1 - \beta) (1 - \beta + 2\beta\kappa + \kappa^2 - 2\beta\kappa^2)}{1 + \kappa^2 - \beta (1 - \kappa + \kappa^2)},
\]  

(62)

- actual return on deposits and limits on withdrawals are \( r_t^b = -1 \) for all \( b \) and \( l_t^b = 0 \) for all \( h \).

In \( t + 1 \), new banks enter the banking market and the economy reverts to normal. Thus, the bankless crisis equilibrium cannot be extended to last more than one period, unless further restrictions are added such as restrictions to entry in the banking market and restrictions to the amount of deposits that a bank can manage.

Also, note that restrictions on parameters can be added such that \( Q_t \) in Proposition D.1 is lower than the good-equilibrium price \( Q^* \). The resulting restrictions can be verified to hold numerically (I have not found any combination of parameters for which they are not satisfied).

**Proof.** Using (5) and the assumptions of the Proposition about the state of the economy, then the net worth of both active and exiting banks is negative. Given \( R_t^D - R_t^K \), choices of banks follow from the non-negativity constraint on money and capital, from \( N_t^b < 0 \), and footnote 8. The actual return on deposits and the limits on withdrawals follow from the fact that no bank is active, and the non-negativity constraints on money and capital are binding as discussed above.

The choice \( D_t^h = 0 \) (and thus \( \eta_t^D = 0 \)) follows trivially from households’ beliefs. Thus, using equation (56) (from the proof of Proposition 3.3, see Appendix C), household beliefs (that trivially satisfy rationality), and the constraint \( \eta_t^M + \eta_t^K + \eta_t^K - 1 \) (thus \( \eta_t^K - 1 - \eta_t^M \)):

\[
\max_{\eta_t^M} \left\{ \kappa \xi \log \left( \frac{\eta_t^M}{P_t} \right) + \kappa \frac{1 - \beta}{1 - \beta} \log \left( \left( 1 - \eta_t^M \right) \left( 1 + R_t^K \right) \right) + \right. \\
\left. \left( 1 - \kappa \right) \frac{\beta}{1 - \beta} \log \left( \left( 1 - \eta_t^M \right) \left( 1 + R_t^K + \eta_t^M \right) \right) \right\}
\]
subject to $\eta^M_n^b, \eta^P_n^b, \eta^K_n^b \in [0, 1]$. The variable $\eta^M_n^b$ solves the FOC:

$$\frac{1}{\eta^M_n^b} - \kappa \frac{\beta}{1 - \beta} \frac{1}{1 - \eta^M_n^b} - (1 - \kappa) \frac{\beta}{1 - \lambda} \frac{R^K_n^b}{(1 + R^K_n^b)} + \eta^M_n^b = 0$$

and using the nominal interest rate (4), the price $Q_t$ in the statement of the Proposition, and $Q^*$ from definition A.4, equation (62) follows. Since $D^K_0 = 0$, $w^K_0 = 0$ from the constraint (16).

Dividends follow from equation (31) and the fact that $N^*_s < 0$ for all $s \in S$.

The market clearing condition for money (34) requires $\int_0^1 M^b_t dh = \overline{M}$, which holds because the wealth of the representative household is $\overline{A}_t = \overline{K}Q_t + \overline{M}$ (banks hold no capital and no money, and $\pi_t = 0$) and $\eta^K_0 \overline{A}_0 = \overline{M}$ where the equality follows using (62). Market clearing for deposits also holds trivially. Market clearing for goods follows from the fact that only a fraction $\kappa$ of households (those who are impatient) buy consumption goods at night (and therefore consumption expenditure is $\kappa \overline{M}$). The total demand for goods is thus $\Delta \overline{M} = \Delta \overline{K}$ (and it is thus equal to the supply) where the equality follows using $p_t$ in the statement of the Proposition. The market clearing for capital holds by Walras’ Law.

Finally, I need to show that $Q_{t+1} = Q^*$ in order to show that $R^K_t$ satisfies (4). This is the case because, in $t + 1$, the state variables of active banks are $X^K_{t+1} = \{(0, 0, 0, 0)\}$ (because new banks with no previous assets and liabilities enter the market). Thus the conditions of Proposition A.8 are satisfied.

\[\square\]

\section{Bad equilibrium}

Consider the state variables of banks $\{X^K_t\}_{t \in [0, 1]}$ described in Proposition A.7. A fraction $\alpha$ of banks is hit by a negative shock $\psi$, and the remaining fraction $1 - \alpha$ is hit by $\psi$. I conjecture that, at night, households run on banks hit by $\psi$, and then I solve for the “candidate equilibrium” using the non-linear system of equations described below. The conjecture is verified if $r_t(\psi) < 0$ so that “running” is indeed the optimal choice of households (see Proposition 3.3) and the “candidate equilibrium” is an equilibrium. The initial conjecture is confirmed for a wide range of parameters. For some values of the parameters, $0 < r_t(\psi) < R^K_t$ so there exists no bad equilibria with runs on a fraction of the banking system for those parameters.

To describe households beliefs during the day, note that household $h \in \mathbb{H}$ holding deposits at bank $b(h)$ faces one of the following possibilities at night.

1. With probability $1 - \alpha$, bank $b(h)$ pays the promised return $R^K_t > 0$. Therefore, bank $b(h)$ is not subject to runs, and household $h$ can withdraw any amount $\leq D^K_h$.
2. With probability $\alpha$, bank $b(h)$ is bankrupt in $t + 1$, paying a return $r_t(\psi) < 0$. Therefore the
Table 2: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.988</td>
</tr>
<tr>
<td>$Z$</td>
<td>Productivity, $y (K) = ZK$</td>
<td>1/3</td>
</tr>
<tr>
<td>$\mathcal{M}$</td>
<td>Money supply</td>
<td>1</td>
</tr>
<tr>
<td>$\mathcal{K}$</td>
<td>Supply of capital</td>
<td>1</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Negative shock to capital</td>
<td>-0.25</td>
</tr>
<tr>
<td>$\overline{\psi}$</td>
<td>Positive shock to capital</td>
<td>0.03</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Fraction of banks hit by the shock $\overline{\psi}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>$\Pr (\varepsilon^h_t = \varepsilon)$</td>
<td>${0.5, 0.85}$</td>
</tr>
</tbody>
</table>

The value of $\varepsilon$ is determined by $\kappa$ and the normalization in equation (2). The value of $\lambda$ is computed using Assumption A.1. The parameters satisfy Assumption A.2.

The optimal choice for household $h$ is to run to try to withdraw as much as possible:

(a) with probability $f_t^{(h)}$, household $h$ is “first in line” ($t^h_t = +\infty$) so it is able to withdraw any amount of money $w_t^h \leq D_t^h$;

(b) with probability $1 - f_t^{(h)}$, household $h$ is “last in line” ($t^h_t = 0$), so it is unable to withdraw money, $w_t^h (\nu_t^h | t^h_t = 0) = 0$. In this case, if household $h$ is impatient ($\varepsilon_h = \varepsilon$), it is able to buy some consumption goods only if it chose to hold some money $M_t^h > 0$ during the day.

Therefore, the rational beliefs of households are:

$$D_t^h \left( r_t^{(h)} - R_t^D, t^h_t = +\infty \right) = 1 - \alpha$$  \hspace{1cm} (63)

$$\Pr_t^{(h)} \left( r_t^{(h)} = r_t (\psi) < 0, t^h_t = +\infty \right) = f_t^{(h)} \alpha$$  \hspace{1cm} (64)

$$\Pr_t^{(h)} \left( r_t^{(h)} = r_t (\psi) < 0, t^h_t = 0 \right) = (1 - f_t^{(h)}) \alpha.$$  \hspace{1cm} (65)

The full set of equations that is used to solve for the equilibrium is described next. The equations refer to the general model with dividends and bankers presented in Appendix A. The equations that refer to the model presented in the main text, Section 2, are a special case that can be obtained by setting $\lambda = 0$.

**Households.** Households hold capital and money that is not held by banks, plus deposits. The total wealth of households $\mathcal{A}_t$ is determined by an expression similar to equation (44) (total value
of capital and money in the economy minus what is held by banks, plus value of deposits):

$$
\bar{A}_t = \left[ \bar{K} - K^* - \lambda \frac{K^*}{1 - \lambda} \right] Q_t + \left[ M - m^* - \lambda \frac{m^*}{1 - \lambda} \right] + \\
+ \lambda \alpha \left[ \frac{K^*}{1 - \lambda} (1 + \psi) Q_t + \frac{m^*}{1 - \lambda} \right] + (1 - \lambda \alpha) d^*. \tag{66}
$$

The first term is the value of capital: the total supply $\bar{K}$ minus capital owned by active banks $K^*$ (defined by equation 36) minus capital owned by banks under liquidation $\lambda \frac{K^*}{1 - \lambda}$. The second term is the value of money; similarly to capital, it is given by the total supply of money $M$ minus the money owned by active banks $m^*$ (defined by equation 37) minus the money held by banks under liquidation $\lambda \frac{m^*}{1 - \lambda}$. The third term is the value of deposits from banks hit by the exit shock and by $\psi$ (such banks are insolvent, so all their assets are used to repay depositors). The last term is the value of deposits at other banks.

The household problem (14) can be rewritten:\(^{35}\)

$$
\max_{\eta_t^M, \eta_t^D, \eta_t^K} \left\{ (1 - \alpha) \left[ \kappa \varepsilon \log (\eta_t^M + \eta_t^D) + \frac{\beta}{1 - \beta} \kappa \log (\eta_t^K) \right] + \frac{\beta}{1 - \beta} (1 - \kappa) \log \left( \eta_t^K (1 + R_t^K) + \eta_t^D (1 + R_t^D) + \eta_t^M \right) \right. \\
+ \alpha (1 - f_t) \left[ \kappa \varepsilon \log (\eta_t^M + \eta_t^D) + \frac{\beta}{1 - \beta} (\kappa \log \eta_t^M + (1 - \kappa) \log \left( \eta_t^K (1 + R_t^K) + \eta_t^D (1 + R_t^D) + \eta_t^M \right) \right] \\
+ \alpha (1 - f_t) \left[ \kappa \varepsilon \log \eta_t^M + \frac{\beta}{1 - \beta} \kappa \log \left( \eta_t^K (1 + R_t^K) + \eta_t^D (1 + R_t^D) + \eta_t^M \right) \right] \\
+ \frac{\beta}{1 - \beta} (1 - \kappa) \log \left( \eta_t^K (1 + R_t^K) + \eta_t^D (1 + R_t^D) + \eta_t^M \right) \right\}
$$

subject to $\eta_t^M + \eta_t^D + \eta_t^K \leq 1$. Using:

$$
\eta_t^K = 1 - \eta_t^M - \eta_t^D, \tag{67}
$$

\(^{35}\) I use the derivation in the proof of Proposition 3.3, in Appendix C, in particular equation (56), combined with the beliefs specified by equations (63), (64), and (65). The last line of equation (56) encompasses the cases $(\varepsilon, r_t^{b|h}, \psi) \in \{(0, R_t^D, +\infty), (0, r_t, \psi), 0\}$. 

the FOCs with respect to $\eta^M$ and $\eta^K$ are:

\[
(1 - \alpha) \left[ \frac{1}{\eta^D + \eta^M} - \frac{1}{1 - \beta} \left( \frac{R^D - R^K}{1 - \eta^M - \eta^K} + \frac{R^D - R^K}{1 + (1 - \eta^M) R^K + \eta^K (R^D - R^K)} \right) \right] \\
+ \alpha f_t \left[ \frac{1}{\eta^D + \eta^M} - \frac{1}{1 - \beta} \left( \frac{R^D - R^K}{1 - \eta^M - \eta^K} + \frac{R^D - R^K}{1 + (1 - \eta^M) R^K + \eta^K (R^D - R^K)} \right) \right] \\
+ \alpha (1 - f_t) \left[ \frac{1}{\eta^M - \eta^K} - \frac{1 - \beta}{1 - \beta} \left( 1 - \eta^M (1 + R_t) + \eta^K (R^b_t - R^K) \right) \right] \\
- \frac{\beta (1 - \alpha)}{1 - \beta} \left( R^K_t + \eta^K (R^b_t - R^K) \right) = 0 \tag{68}
\]

\[
(1 - \alpha) \left[ \frac{1}{\eta^D + \eta^M} - \frac{1}{1 - \beta} \left( \frac{(1 - \alpha) (R^D - R^K)}{1 - \eta^M - \eta^K} + \frac{(1 - \alpha) (R^D - R^K)}{1 + (1 - \eta^M) R^K + \eta^K (R^D - R^K)} \right) \right] \\
+ \alpha f_t \left[ \frac{1}{\eta^D + \eta^M} - \frac{1}{1 - \beta} \left( \frac{R^D - R^K}{1 - \eta^M - \eta^K} + \frac{R^D - R^K}{1 + (1 - \eta^M) R^K + \eta^K (R^D - R^K)} \right) \right] \\
+ \alpha (1 - f_t) \left[ \frac{\beta}{1 - \beta} \left( 1 - \eta^M (1 + R_t) + \eta^K (R^b_t - R^K) \right) \right] \\
+ \frac{\beta (1 - \alpha)}{1 - \beta} \left( R^K_t + \eta^K (R^b_t - R^K) \right) = 0 \tag{69}
\]

I numerically verify that $\eta^M$, $\eta^K$, and $\eta_t^D$ lie in the interval $[0, 1]$. The behavior of households is thus described by equations (66), (67), (68), and (69) that determine, given prices, $A_t$, $\eta^K_t$, $\eta^M_t$, and $\eta_t^D$.

**Return on capital and return on deposits.** The return on capital is given by equation (4) evaluated at $Q_{t+1} = Q^*$ (see Proposition A.8 and the discussion thereafter). The return on deposits is $R^K_t - R^K_t$, satisfying the market clearing condition for deposits. Thus I have two equations that determine $R^K_t$ and $R^K_t$.

**Banks.** The net worth of solvent banks $N_t(\psi)$ and insolvent banks $N_t(\psi)$ is computed using equation (5) evaluated at the respective values of $\psi_t$ and using the values of capital $K^*$, money $m^*$, and deposits $d^*$ from equations (36), (37), and (38). I then use the budget constraint of banks (7) separately for good and bad banks, evaluated at the optimal choice of money described by Proposition 3.1 and taking as given the demand of deposits by households (using $R^K_t - R^K_t$, banks
are indifferent about the amount of deposits). The behavior of banks is thus described by four equations (the two definitions of net worth and the two budget constraints) that determine, given prices, $N_t(\tilde{\psi})$, $K_t(\tilde{\psi})$, and $K_t(\tilde{\psi})$.

**Actual return on deposits of insolvent banks and depositors served during a run.** The actual return on deposits of insolvent banks, $r_t(\tilde{\psi})$ is given by equation (11), using $Q_{t+1} = Q^*$ and $\psi_t = M_t^h = \kappa D_t^h$. The fraction of depositors served during a run is given by (12) evaluated at $D_t^h = \eta_t^{\text{P}} \tilde{A}_t$ and $M_t^h = \kappa D_t^h = \kappa \eta_t^{\text{P}} \tilde{A}_t$ that imply $f_t = \kappa$. Therefore I have two equations that determine $\tau_t(\tilde{\psi})$ and $f_t$.

**Dividends.** Using the state variable of banks hit by the exit shock, described in Proposition A.7, the fact that there is a mass $\lambda$ of such banks, and the expression for dividends in equation (31):

$$\pi_t = \lambda (1 - \alpha) \frac{K^* (1 + \tilde{\psi}) Q_t + m^* - d^*}{1 - \lambda} = \lambda (1 - \alpha) \frac{N_t(\tilde{\psi})}{1 - \lambda}$$

because a fraction $\alpha$ of banks is hit by $\tilde{\psi}$ and has negative net worth (thus, all the assets of such insolvent banks are used to repay depositors, and banks hit by $\tilde{\psi}$ do not contribute to dividends).

**Market clearing.** Withdrawal and consumption decisions of households (Proposition 3.3) together with the market clearing condition for goods, equation (35), imply:

$$2 \tilde{A}_t p_t = \pi_t + \kappa \left[ (1 - \alpha) (M_t^h + D_t^h) + \alpha f_t (M_t^h + D_t^h) + \alpha (1 - f_t) M_t^h \right]$$  \hspace{1cm} (70)

where $M_t^h = \eta_t^{\text{M}} A_t^h$, $D_t^h = \eta_t^{\text{P}} A_t^h$, and I consider the wealth $A_t^h - \tilde{A}_t$ of the representative household. Equation (70) says that the total consumption expenditure is equal to dividends (bankers spend all their dividends $\pi_t$ to buy consumption) plus the consumption expenditure of the fraction $\kappa$ of households that are impatient. A fraction $1 - \alpha$ of impatient households deal with solvent banks and are able to withdraw $D_t^h$, so their consumption expenditure is $M_t^h + D_t^h$. A fraction $\alpha f_t$ of impatient households are first in line during runs, so they can withdraw and they spend $M_t^h + D_t^h$ as well. A fraction $\alpha (1 - f_t)$ are last in line during a run and consume using only money $M_t^h$. The market clearing condition for money during the day is given by equation (34) where $\int M_t^h db = \kappa \int D_t^h db = \kappa \eta_t^{\text{P}} \tilde{A}_t$ and $\int M_t^h dh = \eta_t^{\text{P}} \tilde{A}_t$.

The two market clearing conditions for goods and money determine the price level $p_t$ and the price of capital $Q_t$.

**Solution method.** I obtain a non-linear system of 15 polynomial equations with 15 unknowns. Since I take as given the state of the economy $X_t$ and the price of capital $Q_{t+1} = Q^*$ in $t + 1$. 


I am just solving a static problem, for period $t$. I solve the system in Mathematica using the command NSolve and selecting the real solutions that satisfy all the non-negativity constraints on money, deposits, and capital imposed on the maximization problem of households and banks, and the constraints $\eta^M_t, \eta^D_t, \eta^K_t \in [0, 1]$. I use the values of parameters shown in Table 2 and the initial conditions for banks in equations (36), (37), and (38).

The command NSolve in Mathematica computes the numerical Gröbner bases associated with the system of polynomial equations and then uses eigensystem methods to extract numerical roots, finding all solutions to the system. See Kubler and Schmedders (2010) for an introduction to Gröbner bases applied to the computation of equilibria in economic models, and Lichtblau (2000) for a detailed description of the solution method of NSolve.

**F Temporary monetary injections: model**

When the central bank injects money into the economy, some details of the model are slightly different from what is described in the main text of the paper. In this Appendix, I provide the details about households, banks, and market clearing conditions, and I discuss the solution method in the case of temporary monetary interventions (i.e., under the assumption that the central bank chooses money supply $M_{t+1} = \bar{M}$ at $t + 1$). I build on the formulation with dividends and bankers presented in Appendix A (recall that the model of the main text, Section 2, can be obtained as a special case of the model of Appendix A setting $\lambda = 0$).

**F.1 Households**

The Bellman equation is:

$$
V_t(A^h_t) = \max_{M^h_t, D^h_t, K^h_t} \mathbb{E}_t \left\{ \max_{w_x(n^h_t) \in h(n^h_t)} \left[ \varepsilon^h_t \log c^h_t(n^h_t) + \beta \mathbb{E}_t V_{t+1}(A^h_{t+1}(n^h_t, \psi^h_{t+1})) \right] \right\}
$$

subject to the budget constraint (15), the limit on withdrawals (16), the cash-in-advance constraint (17), and a non-negativity constraint on money $M^h_t \geq 0$, deposits $D^h_t \geq 0$, and capital $K^h_t \geq 0$.

Differently from Section 3.2, the value of wealth $A^h_{t+1}(n^h_t, \psi^h_{t+1})$ is:

$$
A^h_{t+1}(n^h_t, \psi^h_{t+1}) = [K^h_t (1 + \psi^h_{t+1})] Q_{t+1} + d^h_t(n^h_t) + m^h_t(n^h_t) + \frac{A^h_t}{A_t} T_{t+1}
$$

where $d^h_t(n^h_t)$ and $m^h_t(n^h_t)$ are defined by equations (19) and (20); $A_t = \int_a A_t dh$ is the total wealth of the household sector; and $T_{t+1}$ are transfers from the central bank to households in $t + 1$ (defined in Appendix F.4). The formulation of equation (72) implies that transfers from the
central bank are distributed proportionally to the wealth $A^h_t$ of each household $h$. This assumption allows me to still be able to guess-and-verify the value function, and to guess-and-verify that policy functions for choices during the day are proportional to wealth.

### F.2 Banks

During the day of time $t$, banks choose money $M^b_t$, deposits $D^b_t$, capital $K^b_t$, and loans from the central bank $L^b_t$. Bank $b$ receives loans from the central bank during the day of time $t$. During the day of $t+1$, bank $b$ has to pay back $L^b_t\left(1 + R^{CB}_t\right)$ where $R^{CB}_t$ is the nominal interest rate charged by the central bank for the loan.

The problem of bank $b$ with net worth $N^b_t$ is:

$$
\max_{D^b_t, L^b_t, M^b_t, K^b_t} \mathbb{E}_t \left[ \max \{0, K^b_t \left(1 + \psi^b_{t+1}\right) Q_{t+1} + m^b_t - d^b_t\} \right]
$$

subject to the budget constraint:

$$
K^b_t Q_{t+1} + M^b_t \leq D^b_t + L^b_t + N^b_t
$$

where $m^b_t$ and $d^b_t$ are the nominal values of money and deposits at the end of the night of time $t$:

$$
m^b_t = (M^b_t - w^b_t) + y (K^b_t) \rho_t, \quad d^b_t = (D^b_t - w^b_t) \left(1 + R^b_t\right) + L^b_t \left(1 + R^{CB}_t\right).
$$

Banks must also satisfy non-negativity constraints on money $M^b_t \geq 0$, deposits $D^b_t \geq 0$, and capital $K^b_t \geq 0$. For simplicity, I focus on the case of a non-negativity constraint on loans too, $L^b_t \geq 0$, even if the central bank might decide to allow for negative loans (the constraint $L^b_t \geq 0$ will not be binding in equilibrium).

The results of Proposition 3.1 about $D^b_t$ and $M^b_t$ are unchanged; in particular, the bank still wants to hold an amount of money $M^b_t = \kappa D^b_t$ because at night it will face withdrawals only from depositors (none of the funds lent by the central bank will be withdrawn at night). The choice of capital $K^b_t$ is $K^b_t = (N^b_t + L^b_t + D^b_t - M^b_t) / Q_t$ and it follows from the budget constraint (74). The choice of $L^b_t$ is similar to the choice of $D_t^b$ and can be proven similarly:

$$
L^b_t = \begin{cases} 
0 & \text{if } R^{CB}_t > R^K_t \\
\text{any amount} & \text{if } R^{CB}_t = R^K_t \\
+\infty & \text{if } R^{CB}_t < R^K_t.
\end{cases}
$$

Intuitively, banks invest all the funds that they get from the central bank in capital (because the
central bank does not withdraw any money at night). Therefore bank $b$ wants to hold $L_b^h = 0$ if the return $R_t^{CB}$ is greater than $R_t^K$, $L_b^h = +\infty$ if $R_t^{CB} < R_t^K$, and any amount of loans if $R_t^{CB} = R_t^K$.

In equilibrium $R_t^{CB} = R_t^K$, to make sure that banks are willing to take the amount of loans offered by the central bank.

### E.3 Actual return on deposits and fraction of depositors served

Under the relevant case $R_t^D = R_t^{CB}$, the actual return on deposits is:

$$r_t^D = \min \{ R_t^D, r_t^K \}.$$  

If loans to banks are senior to deposits, $\tilde{r}_t^D$ solves:

$$E_\theta \{ K_t^b \left( 1 + \psi_{t+1}^b \right) Q_{t+1} \} + ZK_t^b p_t = \left( D_t^b - w_t^b \right) \left( 1 + \tilde{r}_t^D \right) + L_t^b \left( 1 + R_t^{CB} \right)$$  \hspace{1cm} (76)

If loans to banks have the same seniority as deposits, $\tilde{r}_t^D$ solves:

$$E_\theta \{ K_t^b \left( 1 + \psi_{t+1}^b \right) Q_{t+1} \} + ZK_t^b p_t = \left( D_t^b - w_t^b + L_t^b \right) \left( 1 + \tilde{r}_t^D \right)$$  \hspace{1cm} (77)

Using $\psi_{t+1}^b = 0$ with probability one and rearranging, equations (76) and (77) become, respectively:

$$1 + \tilde{r}_t^D = \frac{K_t^b (Q_{t+1} + Z p_t) - L_t^b \left( 1 + R_t^{CB} \right)}{D_t^b - w_t^b}$$  \hspace{1cm} (78)

and:

$$1 + \tilde{r}_t^D = \frac{K_t^b (Q_{t+1} + Z p_t)}{D_t^b - w_t^b + L_t^b}$$  \hspace{1cm} (79)

The fraction of depositors served during the run is given by (12).

### F.4 Central bank

During the day of time $t$, the central bank injects money $\mu_t M$ in the economy by offering loans $L_t^{CB}$ to banks and/or buying assets $K_t^{CB}$ on the market at price $Q_t$. Therefore:

$$M_t = L_t^{CB} + Q_t K_t^{CB}$$  \hspace{1cm} (80)

In $t + 1$, the money supply goes back to the pre-crisis level $M$. The central bank obtains a return $T_{t+1}$ from loans and from holding assets, which is distributed to households as described in
Appendix F.1. If loans to banks are senior to deposits, then:

$$T_{t+1} = L_t^{CB} R_t^{CB} + Q_t K_t^{CB} R_t^K.$$  \hspace{1cm} (81)

Otherwise, if loans to banks have the same seniority as deposits:

$$T_{t+1} = L_t^{CB} \left[ (1 - \alpha) R_t^{CB} + \alpha r_t (\psi) \right] + Q_t K_t^{CB} R_t^K.$$  \hspace{1cm} (82)

A fraction $\alpha$ of loans are given to banks that turn out to be insolvent (because they were hit by the shock $\psi$) and are going bankrupt; therefore the central bank gets a return $r_t (\psi)$ on such loans. The remaining fraction $1 - \alpha$ is given to good solvent banks, and therefore the return on such loans is $R_t^{CB}$. The return on capital $K_t^{CB}$ is the market return $R_t^K$.

F.5 Market clearing conditions

The market clearing conditions (21) and (34) are replaced by:

$$\int_B K_t^d db + \int_{\mathbb{H}} K_t^h dh + K_t^{CB} = K$$  \hspace{1cm} (83)

$$\int_B M_t^h db + \int_{\mathbb{H}} M_t^h dh = M (1 + \mu_t) - \pi_t.$$  \hspace{1cm} (84)

The market clearing conditions for deposits (23) and for consumption goods (35) are unchanged. Additionally, the following condition must hold:

$$I_t^{CB} = \int_B I_t^d db$$  \hspace{1cm} (85)

requiring that the total supply of loans by the central bank is equal to the demand by private banks.

F.6 Equilibrium definition

Definition F.1. Given the initial state of the economy $X_t$, an equilibrium is a collection of:

- prices $Q_t$ and $p_t$ and return on capital $R_t^K$, on deposits $R_t^D$, and on loans by the central bank $R_t^{CB}$;
- household beliefs $D_t^h (\cdot)$ about $r_t^{b(h)}$ and $l_t^h$, for all $h \in \mathbb{H}$;
- household choices $\left\{ M_t^h, D_t^h, K_t^h, \left\{ u^h (r_t^h), c^h (r_t^h) \right\}_{r_t^h \in N} \right\}$ for all $h \in \mathbb{H}$;
- bank choices $\left\{ D_t^h, M_t^h, K_t^h, L_t^h \right\}$ for all $b \in B$;
- limits on withdrawals $l_t^b \in \{0, +\infty\}$ for all $h \in \mathbb{H}$;
- liquidation returns $r_t^b$ and fraction of depositors served during a run $f_t^b$, for all $b \in B$;
• dividends $\pi_t$ paid to bankers;
• central bank: money injected $\mu_t$, loans $L_t^{CB}$, assets purchased $K_t^{CB}$, and transfers to depositors $T_{t+1}$;

such that:
• (banks: optimality, returns, and limits on withdrawals) banks solve problem (73); $r_t^h$ satisfies equation (78) (if loans $L_t^h$ are senior to depositors) or equation (79) (if loans $L_t^h$ have the same seniority as deposits); $f_t^h$ satisfies equation (12); the limit on withdrawals $l_t^h$ of depositor $h \in \mathbb{H}(h)$ of bank $b$ satisfies:

$$l_t^h = 0 \text{ for some } h \in \mathbb{H}(h) \rightarrow \int_{\mathbb{H}(h)} u^h(t_{t+1} \mid t_t^h = +\infty) \, dh > M_t^h;$$

• (households’ optimality) households solve problem (71) and Assumption 3.2 holds;
• (central bank) policies of the central bank satisfy equation (80) (if loans $L_t^h$ are senior to deposits) or equation (81) (if loans $L_t^h$ have the same seniority as deposits);

• (rational expectations) households’ beliefs are rational:

$$Pr_t^h\left(\tau_{x_t}^{(h)} = \tau, t_t^h = l\right) = Pr_t^h\left(\tau_{x_t}^{(h)} = \tau, t_t^h = l\right); \quad \tau \in \mathbb{R} \text{ and } l \in \{0, +\infty\};$$

• (dividends) $\pi_t$ satisfies (31);
• (market clearing) the market clearing conditions hold.

Note that there is no need to include transfers $T_{t+1}$ in the state of the economy in $t+1$. This is because, in $t+1$, the supply of money and capital is, respectively, $\bar{M}$ and $\bar{K}$, and I can compute the wealth of households $\bar{A}_{t+1}$ using the total supply of money and capital minus the value of assets held by banks and bankers, that are included in the state $X_{t+1}$. I focus on symmetric equilibria in which banks with the same state variables make the same choices.

### E7 Numerical solution

I follow the same approach described in Appendix E.

• The FOC of the household problem (68) and (69) are replaced by the expressions derived after guessing-and-verifying the form of the value function of the problem (71); the policy functions for day choices are of the form $M_t^h - \eta_t^M A_t^h, D_t^h - \eta_t^D A_t^h$, and $K_t^h - \eta_t^K A_t^h$.

• The returns on capital and on deposits are unchanged.

• The definition of net worth of banks is unchanged. The budget constraint (7) is replaced by (74) using the fact that all banks get the same amount of loans, $L_t^b = L_t^h$ for all $h \in [0, 1]$ and
using (80) and the supply of loans from (85).
- The actual return on deposits of insolvent banks (10) is replaced by (78) or by (79) depending on the policy; the fraction of depositors served during a run $f_i$ is unchanged, $f_i = \kappa$.
- The expression for dividends is unchanged.
- Equation (70) is unchanged. The market clearing condition (22) is replaced by (84); the market clearing for capital (83) is omitted due to Walras’ Law.
- In addition to the equations described in Appendix E, I include also equation (82) that determine the value of transfers $T_{t+1}$.

## G Temporary monetary injections: additional results

### Additional results for $\kappa = 0.5$. 
Figures 7 and 8 plot the results of temporary monetary injections implemented with asset purchases (analyzed in Section 5.1) and with loans to banks (analyzed in Section 5.2), respectively.

### Results for $\kappa = 0.85$. 
The results plotted in Figures 7 and 8 and derived under Assumption 5.1 are robust to the choices of other parameter values, with a caveat. For some values of the parameters, a bad equilibrium does not exist for some $\mu_t$. Figures 9 and 10 plot, respectively, the effects of asset purchases and the effects of loans to banks with the same seniority as deposits using the parameters in Table 2 and $\kappa = 0.85$.

In the case of asset purchases, a bad equilibrium does not exist for some values of $\mu_t$. The difference between this Appendix and Section 5 is the value of $\kappa$, which represents the fraction of households hit by the high preference shock $\varepsilon$ (by the normalization implied by (1) and (2), $\kappa$ affects also the value of $\varepsilon$). The parameter $\kappa$ governs the stochastic process of the preference shocks, and thus it affects the precautionary demand for money (in the limit in which $\kappa = 1$, all households are impatient and there is no uncertainty about desired consumption expenditure at night). The assumption in equation (1) imposes a restriction on the stochastic process that governs the preference shocks ($\varepsilon = 0$, only two values in the support of $\varepsilon^s_1$), mainly for tractability. Thus, bad equilibria exist despite this restriction on $\varepsilon^s_1$. Considering a less restrictive process for $\varepsilon^s_1$ is likely to reinforce the existence of a bad equilibrium, but verifying this conjecture is a quantitative question that is beyond the scope of this paper.

### Relaxing Assumption 4.1. 
If Assumption 4.1 holds but Assumption 5.1 does not hold (i.e., equation (26) holds with strict inequality), the results of monetary policy analysis are similar, provided that the left-hand side of equation (26) is not too large.
Figure 7: Effects of monetary policy - Asset purchases ($\kappa = 0.5$)

Price of capital $Q_t$

- Mild Crisis
- Deep Crisis

Price level $p_t$

- Mild Crisis
- Deep Crisis

Money held by households $\int M_t^b dh$

- Mild Crisis
- Deep Crisis

Deposits $D_t^b$

- Mild Crisis
- Deep Crisis

Nominal return on capital $R_t^K$

- Mild Crisis
- Deep Crisis

Return on deposits, insolvent banks $r_t (\omega)$

- Mild Crisis
- Deep Crisis

The variable on the horizontal axis is the size of the monetary injection $\mu_t$. For each subplot, the blue line represents the mild crisis equilibrium, the red line the deep crisis equilibrium, and the green dotted line the good equilibrium without monetary intervention. Parameter values: see Table 2, $\kappa = 0.5$. 
Figure 8: Effects of monetary policy - Loans to banks ($\kappa = 0.5$)

Price of capital $Q_t$

Price level $p_t$

Money held by households $\int M_t^h dh$

Deposits $D_t^h$

Nominal return on capital $R_t^h$

Return on deposits, insolvent banks $r_t(\psi)$

The variable on the horizontal axis is the size of the monetary injection $\mu_t$. In each subplot, the green dashed line represents the good equilibrium without any monetary intervention, the blue solid line the equilibrium value of the endogenous variables for which the monetary injection $\mu_t$ results in $r_t(\psi) < 0$, and the blue dotted line the candidate bad equilibrium for values of $\mu_t$ such that $r_t(\psi) > 0$ (see Section 4.2). Parameter values: see Table 2, $\kappa = 0.5$. 
Figure 9: Effects of monetary policy - Asset purchases ($\kappa = 0.85$)

Price of capital $Q_t$

Price level $p_t$

Money held by households $\int M^h_t dh$

Deposits $D^b_t$

Nominal return on capital $R^K_t$

Return on deposits, insolvent banks $r_t(\psi)$

The variable on the horizontal axis is the size of the monetary injection $\mu_t$. For each subplot, the blue line represents the mild crisis equilibrium, the red line the deep crisis equilibrium, and the green dotted line the good equilibrium without monetary intervention. Parameter values: see Table 2, $\kappa = 0.85$. 
The variable on the horizontal axis is the size of the monetary injection $\mu_t$. In each subplot, the green dashed line represents the good equilibrium without any monetary intervention, the blue solid line the equilibrium value of the endogenous variables for which the monetary injection $\mu_t$ results in $r_t(\psi) < 0$, and the blue dotted line the candidate bad equilibrium for values of $\mu_t$ such that $r_t(\psi) > 0$ (see Section 4.2). Parameter values: see Table 2, $\kappa = 0.85$. 
If the left-hand side of (26) is above a certain threshold (that depends on the parameters of the model), then banks hit by the bad shock have enough net worth to absorb the losses due to the drop of $Q_t$ in the bad equilibrium without becoming insolvent. Therefore, no bank becomes insolvent and no bad equilibrium exists.

For small values of the left-hand side of equation (26), the results are qualitatively identical to Section 5 using the relevant threshold for $Q_t$ (see discussion in Section 5).

For intermediate values of the left-hand side of equation (26), the results are similar to Section 5 but a bad equilibrium does not exist for some values of $\mu_t$. For instance, for $\kappa = 0.5$, if $K^*_{t-1}$ and $m^*_{t-1}$ are 5% higher than the values of $K^*$ and $m^*$ defined in equations (36) and (37), the threshold for $Q_t$ is about 76; the mild crises equilibrium exists for $\mu_t < 0.15$; the deep crisis equilibrium exists for $\mu_t \in (0.55, 0.8)$ (and $\mu_t \geq 0.8$ results in $Q_t \geq 76$); and no bad equilibrium exists for $\mu_t \in [0.15, 0.55]$. This result is due to the fact that, for $\mu_t > 0.15$, the actual return of insolvent banks $r_t(\psi) > 0$ and thus the candidate mild crisis equilibrium is not an equilibrium (see Section 4.2), and for $\mu_t < 0.55$ the constraint (27) is not satisfied (see Section 4.4).

H Permanent monetary injections

I analyze a once-and-for-all increase in the money supply, setting $M_{t+j} = \bar{M} (1 + \mu)$ for all $j \geq 0$ (the non-zero shocks to capital hit the economy at time $t$). The once-and-for-all increase in the money supply does not capture any interventions used during the recent financial crisis, but it represents a useful benchmark. The money is distributed lump-sum to households at the beginning of time $t$ (the initial stock of money of household $h$ is $m^*_{t-1} + \mu \bar{M}$). Similar results hold in the case of asset purchases and loans to banks.

A sufficiently large increase in the money supply, represented by a sufficiently high $\mu$, eliminates the bad equilibria. From equation (4), recall that the nominal return on capital is:

$$1 + R^K_t - \frac{Q_{t+1} + Zp_t}{Q_t}.$$ 

Differently from temporary monetary injections analyzed in Section 5, a permanent monetary injection results in an increase of both $Q_t$ and $Q_{t+1}$. In the numerical example that I consider, $R^K_t$ is approximately constant, independent of the size of the monetary injection $\mu$. As a result, the policy dampens the flight to liquidity, and deposits increase. Moreover, due to the increase of $Q_{t+1}$, the return $r_t(\psi)$ paid by insolvent banks goes up as well. The higher $Q_{t+1}$ and a reduced flight to liquidity allow to eliminate the bad equilibria with a moderate monetary intervention. The results are presented in Figure 11.
Figure 11: Permanent monetary injection

The variable on the horizontal axis is the size of the monetary injection $\mu_t$. In each subplot, the green dashed line represents the good equilibrium without any monetary intervention, the blue solid line the equilibrium value of the endogenous variables for which the monetary injection $\mu_t$ results in $r_t(\psi) < 0$, and the blue dotted line the candidate bad equilibrium for values of $\mu_t$ such that $r_t(\psi) > 0$ (see Section 4.2). Parameter values: see Table 2, $\kappa = 0.5$. 
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