"Breaking the Sovereign-Bank Nexus"

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Breaking the Sovereign-Bank Nexus*

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Abstract

This paper develops a quantitative dynamic general equilibrium model that features endogenous bank failure and sovereign default risk. It studies the feedback loop between sovereign and banking crises, and evaluates the effectiveness of bank capital regulation in addressing it. In the model, bank failure contributes to an increase of sovereign default risk through the government bailout of bank creditors. Meanwhile, holding high-yield risky sovereign bonds may be attractive to banks protected by limited liability. By increasing banks' failure risk and their funding costs, sovereign exposures hurt bank lending and contribute to further contractions in aggregate economic activity. Capital requirements shape banks' incentives to invest in sovereign debt. More stringent capital regulation makes banks safer, weakening the sovereign-bank nexus. This comes at the cost of constraining the overall supply of credit.

Keywords: banking crises, sovereign risk, macroprudential policy, capital regulation.

JEL codes: E44, F34, G01, G21, G28

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“Our challenge in the euro area is to ensure that, when banks fail and the public sector has to intervene, it does not result in a recurrence of the bank-sovereign nexus.”

– Mario Draghi (2014)

1 Introduction

The nexus between sovereign and bank risk, often referred to as the “diabolic loop” (Brunnermeier et al., 2016) or “doom loop” (Farhi and Tirole, 2018), has drawn considerable attention since the onset of the European debt crisis. When the financial health of banks deteriorated as a result of the Global Financial Crisis, the combination of national governments’ support to their domestic banking systems and lower tax revenues put pressure on the public finances of a number of countries. At the same time, the elevated exposure of banks to their domestic sovereign debt translated the weakness of public finances into further weakness for banks. The cost of borrowing for governments, banks, and non-financial companies rose sharply, depressing investment and economic activity, and further amplifying the initial contraction.

In view of this experience, several voices called for changes in the regulatory treatment of banks’ exposures to (domestic) sovereign debt.1 Existing capital regulation imposes that at least a fraction of the banks’ risk-weighted assets has to be financed with bank equity capital. However, as of now, it assigns zero risk weights to domestic sovereign debt. Furthermore, these exposures are also exempt from concentration limits to single counterparties.2

This paper develops a quantitative dynamic general equilibrium model that captures the non-linearities associated with the sovereign-bank nexus and their implications for aggregate economic activity. In the model, banks intermediate funds between households and firms, and hold sovereign bonds for liquidity management purposes. A government provides bailout guarantees on bank liabilities, specifically in the form of (partial) deposit insurance, and places its risky sovereign debt among domestic banks and international investors. The model

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1 For example Brunnermeier et al. (2011) and Bénassy-Quéré et al. (2018).
focuses on the interplay between endogenous bank failure risk and sovereign default risk. The former stems from the exposure of banks to risky private sector assets, as well as to risky sovereign debt. Sovereign default risk is in turn affected by bank risk through the deposit insurance liabilities.

Distortions associated with external debt financing drive the risk-taking incentives of banks and, eventually, the weight of sovereign debt exposures in their balance sheet. Limited liability makes investing in high-yield, risky sovereign debt attractive for banks’ shareholders: they can enjoy high profits insofar as the government does not default, while their losses are limited to their initial equity contribution otherwise. At the same time, deposit insurance and the opacity of banks’ balance sheets precludes depositors from pricing individual bank failure risk at the margin. Bank deposits are priced based on depositors’ expectations about the potential losses associated with the risk of failure of the average bank, rather than the risk-taking decisions of each individual bank. Together, these frictions lead to a risk-shifting channel which encourages excessive leverage and excessive exposure of banks to sovereign risk.

Bank capital regulation determines the minimum amount of equity with which banks need to finance their investments. Limited participation in equity markets, however, constrains the amount of internal equity financing available to banks, which evolves endogenously as a function of retained bank profits. This gives rise to a net worth channel similar to the financial accelerator in Bernanke, Gertler, and Gilchrist (1999) and Gertler and Kiyotaki (2010), which effectively links aggregate economic activity to banks’ balance sheet conditions.

Fluctuations in the model are driven by bank risk shocks, namely, shocks to the cross-sectional dispersion of idiosyncratic risk, similar to those in Christiano, Motto, and Rostagno (2014), which in this model affect banks’ asset returns. When bank risk is elevated, bank failure and the fiscal costs associated with government guarantees surge. Increased debt issuance to finance these costs drives up sovereign default risk and, thus, the borrowing

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Risk shocks (also referred to as uncertainty shocks or volatility shocks) have been shown to be important in driving business cycle fluctuations (Christiano et al., 2014; Bloom et al., 2018) and to play a key role in generating sharp recessions (Bloom, 2009), and financial crises (Arellano, Bai, and Kehoe, forthcoming).
cost for the government. Due to the risk-shifting channel, higher yields make sovereign debt relatively more attractive to banks, which increase their sovereign exposures. This in turn increases banks’ borrowing costs (bank funding cost channel), which erode their profits, reduce their net worth, and tighten the constraints they face (bank net worth channel). The two forces combined result in a higher cost of finance for investment activities and, ultimately, lower aggregate investment and output. Importantly, the mutually reinforcing effects of sovereign and bank risk are transmitted into reduced economic activity even when the sovereign default event does not materialize.

The risk-shifting distortions associated with government guarantees and the implications of deposit insurance and banks’ opacity for the pricing of their failure risk provide a rationale for bank capital regulation, as in Kareken and Wallace (1978). Higher capital requirements can mitigate risk-shifting incentives of banks. By reducing their leverage, banks become safer and their funding costs become lower. When capital requirements are too high, however, equity funding becomes more expensive (due to the relative scarcity of equity at the aggregate level), restricting the size of the banking sector and contracting investment and output.

The model is internally calibrated to match a set of empirical targets for Spain, a large peripheral European economy whose evolution in the aftermath of the Global Financial Crisis fits very well the standard narrative on the sovereign-bank nexus. The calibration allows to capture the dynamics of a number of macroeconomic and financial variables around the events of the European debt crisis. The model is able to reproduce, both in qualitative and in quantitative terms, the increase in sovereign and bank yields, as well as in the exposure of banks to domestic sovereign debt, and the subsequent contraction of credit and output. In particular, sovereign and bank borrowing spreads in the typical crisis in the model follow closely those observed during the sovereign debt crisis in Spain.

After documenting the quantitatively important amplification effects resulting from the presence of the sovereign-bank nexus, the paper explores whether amendments to the existing capital regulation can help mitigate them. For any given reference level of the capital requirement (per unit of risk weighted assets), the introduction of positive risk weights on sovereign exposures reduces banks’ endogenous exposure to sovereign risk and makes them
effectively safer. This is particularly the case when capital requirements are relatively low, and thus bank leverage is high. So if the capital requirement is low, the socially optimal risk weight on sovereign exposures is positive. However, if the policy maker can choose both the level of the capital requirement and the risk weight on sovereign exposures, the optimal policy mix features a higher capital requirement and a zero risk-weight on sovereign debt. This result arises because setting positive risk weights on sovereign debt has the unintended effect of crowding out lending to the non-financial sector during crises: since the aggregate level of equity is fixed in the short run, requiring banks to use part of it to finance their investment in sovereign debt forces them to reduce their investment in other productive assets.

A number of papers have analyzed the underpinnings of the nexus between banks and sovereigns in stylized theoretical frameworks, including Acharya, Drechsler, and Schnabl (2014), Gennaioli, Martin, and Rossi (2014), Brunnermeier et al. (2016), Cooper and Nikolov (2018), Leonello (2018), and Farhi and Tirole (2018). The contribution of this paper is to embed some of the main mechanisms highlighted in previous theoretical work in a dynamic general equilibrium model which is able to quantitatively reproduce the effects of the sovereign-bank nexus, and which is used to assess the potential of capital regulation to mitigate its negative effects.

Some recent papers analyze the interaction between sovereign defaults and bank credit in quantitative macroeconomic setups. Bocola (2016) focuses on the pass-through of sovereign risk to private credit provision in an environment in which exogenous sovereign risk shocks make banks suffer losses on their debt holdings, reducing their net worth and thus constraining credit supply. In Sosa-Padilla (2018) and Perez (2018), the government’s incentive to honor its debt is affected by the awareness that the realization of a sovereign default can generate a sharp contraction in bank credit and economic activity. The papers in this strand of the literature, however, have so far abstracted from bank failure and its connection to government finances via bailout guarantees. In fact, banks in these models obtain external funding at the risk-free rate, which is at odds with empirical evidence, and implies underes-
stimating the credit crunch effects of the nexus.\textsuperscript{4} Modeling bank failure and the distortions associated with it is especially important in order to explore the potential for capital regulation to mitigate the negative effects of the sovereign-bank nexus. This is one of the key goals of this paper.

The link between bank capital and aggregate investment is similar to that explored by Gertler and Kiyotaki (2010), Gertler and Karadi (2011), He and Krishnamurthy (2012), and Brunnermeier and Sannikov (2014). This paper relates to the literature that assesses the effects of bank capital requirements from a macroeconomic perspective, including Angeloni and Faia (2013), Martinez-Miera and Suarez (2014), Clerc et al. (2015), Mendicino et al. (2018, 2019, forthcoming), Elenev, Landvoigt, and Van Nieuwerburgh (2018), Malherbe (forthcoming), and Begenau (forthcoming). None of these papers consider sovereign risk.

Methodologically, this paper relates to recent efforts to solve quantitative models of financial crises using global solution methods.\textsuperscript{5} These papers highlight the importance of non-linear dynamics and time-varying risk premia that traditional local solution methods are not able to capture. These features are particularly relevant in the context of this paper, as sovereign default episodes are inherently non-linear events and default risk causes large variations in risk premia with important consequences for macroeconomic outcomes, as shown below.

The remaining of the paper is organized as follows. Section 2 presents some motivating evidence about the sovereign-bank nexus in the context of the European debt crisis. Section 3 describes the model setup. Section 4 describes the quantitative analysis, including the calibration of the model and its main properties. Section 5 explores some counterfactual exercises about the contribution of sovereign risk as an amplification mechanism, and about the potential effects of capital regulation on the sovereign-bank nexus. Section 6 concludes.

\textsuperscript{4}See Section 2 for empirical evidence on the borrowing costs for banks during the European debt crisis.

\textsuperscript{5}This feature is shared by some of the above-mentioned references including Bocola (2016).
2 Motivating evidence

The link between sovereign and banking crises is not new. Reinhart and Rogoff (2011) and Jorda, Schularick, and Taylor (2016) document it using long historical time series for a wide range of countries. This section documents three main stylized facts about the sovereign-bank nexus observed during the European debt crisis.

Fact 1. During the European debt crisis, interest rate spreads of sovereigns, banks, and corporates in the periphery opened widely, with sizeable heterogeneity across countries. As reported in Figure 1 using spreads data constructed by Gilchrist and Mojon (2018), private costs of borrowing increased during the Global Financial Crisis for both core and periphery countries in Europe (Panel A). However, they started diverging at the onset of the sovereign debt crisis, when sovereign spreads widened in the periphery, reflecting the deterioration in financial conditions and the increase in perceived riskiness of borrowers in these countries.

Almeida et al. (2017) find that increases in the perceived riskiness of sovereign debt (rating downgrades) translate into higher funding costs for banks, while Bahaj (2019) documents the pass-through from higher sovereign spreads into higher funding costs for non-financial companies. Gilchrist and Mojon (2018) document the forecasting power of bank credit spreads for economic activity using data from the period around the European sovereign debt crisis.

Fact 2. During the European debt crisis, government finances in peripheral countries deteriorated significantly and domestic banks substituted for foreign investors in the holding of debt of the most affected sovereigns. While sovereign debt levels increased for both core and periphery countries in Europe during the first phase of the crisis (2009–2010), debt dynamics started to diverge during the second phase (2011–2012), as reported in Figure 1 (Panel B.1). The debt-to-GDP ratio in Spain increased rapidly, going from 40% at the beginning of the crisis to 90% in 2012. In the case of Italy, which started with a higher level, it went from 100% to 130% during the same period.

Banks in peripheral Europe increased their domestic sovereign exposures, while banks in core countries kept theirs relatively constant. Domestic sovereign bond holdings as a fraction
Panel A: Sovereign, bank, and non-financial corporate spreads

A.1: Sovereign spreads  
A.2: Bank spreads  
A.3: Corporate spreads

Panel B: Sovereign debt dynamics and bond holdings

B.1: Debt-to-output ratio  
B.2: Banks’ exposures  
B.3: Foreigners’ holdings

Panel C: Output, investment, and bank lending

C.1: Gross domestic product  
C.2: Investment  
C.3: Bank lending to NFCs

Figure 1: Motivating evidence
of total assets went up by 8 percentage points (pp) in Spain and Italy (Panel B.2). At the same time, the share of bonds held by foreign investors went down substantially (Panel B.3).

Banks’ tendency to increase their holdings of domestic sovereign debt during times of sovereign stress can arise from risk-shifting related distortions, as analyzed in a theoretical setup by Crosignani (2017) and Ari (2018), and documented empirically by Battistini, Pagano, and Simonelli (2014), Acharya and Steffen (2015) and Altavilla, Pagano, and Simonelli (2017) in the context of the European debt crisis.\(^6\) Uhlig (2014) studies the incentives of opportunistic regulators in risky countries within a monetary union to allow their banks to hold domestic bonds as a way of shifting the risk of potential sovereign default losses to safer countries. Gaballo and Zetlin-Jones (2016), in contrast, find that this equilibrium outcome might arise endogenously, preventing government bailouts and thereby imposing discipline on banks.

**Fact 3.** During the European debt crisis, contractions in output, investment and bank lending visibly correlated with the intensity of the sovereign-bank nexus. Peripheral economies in Europe witnessed a “double dip” in output and investment substantially more pronounced than the one in core countries (Panels C.1 and C.2). Particularly severe was the contraction in bank credit to non-financial companies, reflecting the deterioration in the financial condition of banks during this period (Panel C.3).


\(^{6}\)Other reasons for this tendency include creditor discrimination by defaulting governments, which creates a difference between the expected return on sovereign bonds for domestic banks and foreign investors (see Broner, Erce, Martin, and Ventura, 2014), or financial repression (see Acharya and Rajan, 2013, and Chari, Dovis, and Kehoe, forthcoming, for theoretical models reflecting this channel, and Becker and Ivashina, 2017, Altavilla et al., 2017, and Ongen, Popov, and Horen, forthcoming, for related evidence from the European crisis).
3 A model of the sovereign-bank nexus

Time is discrete and runs infinitely. There is a single non-durable consumption good, which is also used as the numeraire and which can be transformed into physical capital used for production. The domestic economy is populated by: (i) an infinitely-lived representative household; (ii) a mass of bankers that run a continuum of banks; (iii) a continuum of capital producers that transform consumption goods into physical capital; (iv) a representative firm that produces consumption goods combining labor and physical capital; and (v) a government partially funded by risky sovereign debt. In addition, there are international investors that invest in the sovereign debt issued by the government. Figure 2 depicts the connections between the balance sheets of the different agents in the economy.

The representative household takes consumption and savings decisions to maximize its intertemporal expected utility. It inelastically supplies labor and can invest its savings in bank deposits (partially) guaranteed by the government and in holding claims on physical capital issued by entrepreneurs.

Bankers are a special class of members of the representative household with exclusive temporary access to the opportunity of investing their net worth as banks’ inside equity capital. Once they become bankers, they accumulate wealth until they retire, when they transfer it to the representative household and are replaced by new bankers.

Banks are perfectly competitive and operate under limited liability. They borrow from households and issue equity among bankers in order to comply with a regulatory capital requirement, which effectively constrains their intermediation ability. They invest both in sovereign debt (from which the obtain some liquidity services complementary to their deposit-taking activity) and corporate claims.

Entrepreneurs need to borrow in order to transform consumption good into physical capital. Physical capital is rented to perfectly competitive firms, which combine it with labor in order to produce consumption good.

The government issues short-term debt to finance its deficit and the cost of the guarantees on bank liabilities. It may (stochastically) default, with a probability that increases in its
level of debt. Default implies the write-off of a fraction of the outstanding debt, resulting in losses to bond holders. Sovereign debt is placed among domestic banks and international investors.

The following subsections describe each of these agents, their optimization problems, and the definition of equilibrium in detail.

### 3.1 Production

A representative, competitive firm combines physical capital $K_t$ and labor $L_t$ to produce a homogeneous good $Y_t$ using a constant returns to scale technology

$$Y_t = K_t^\alpha L_t^{1-\alpha}. \quad (1)$$

Physical capital and labor are rented in competitive markets at rates $r_t^K$ and $W_t$, respectively. Capital depreciates at a rate $\delta$.

Physical capital is produced by some firms with access to a constant returns-to-scale investment technology. This technology requires, as input, an investment of one unit of
consumption good at time \( t \), to produce a stochastic amount of capital \( \omega \) at \( t + 1 \), where \( \omega \) is an idiosyncratic shock independently and identically distributed across firms and across time. These firms finance their investment \( A_t \) by selling claims on the returns of the physical capital that they will produce at \( t + 1 \). The return on each unit of capital effectively produced at \( t + 1 \) is \( R^K_t = r^K_t + 1 - \delta \), that is, the rental rate of capital plus undepreciated capital recovered after production takes place.

### 3.2 Households

The representative household is infinitely lived and, in each period \( t \), obtains utility \( U(C_t) \) from the consumption of non-durable goods \( C_t \), where \( U(\cdot) \) is a standard concave, twice continuously differentiable function. It inelastically supplies one unit of labor remunerated with a wage \( W_t \), receives net dividend payments from banks \( \Pi_t \), and pays lump-sum taxes \( T_t \). The problem of the representative household involves choosing consumption \( C_t \), partially insured deposits \( D_t \), and investment in claims issued by capital-producing firms \( A^h_t \), so as to maximize its expected discounted lifetime utility

\[
E_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}),
\]

subject to the budget constraint:

\[
C_t + D_t + A^h_t + h(A^h_t) = W_t + \tilde{R}^D_t D_{t-1} + R^K_t A^h_{t-1} + \Pi_t - T_t,
\]

where \( \beta \) is the subjective discount rate, and \( \tilde{R}^D_t \) and \( R^K_t \) denote, respectively, gross realized returns on deposits and on claims of capital-producing firms.\(^7\) The realized return on deposits is

\[
\tilde{R}^D_t = R^D_{t-1} - (1 - \chi)\Psi_t,
\]

\(^7\)The household is perfectly diversified across capital-producing firms and banks, and thus the returns on its investments are not affected by idiosyncratic risk.
which amounts to the promised gross interest rate $R_{t-1}^D$ minus the losses realized in case of bank failure. The government insures a fraction $\chi$ of the promised repayments of principal and interest associated with bank deposits $R_{t-1}^D$. The remaining part of those promised repayments is subject to potential losses $\Psi_t$ per unit of deposits derived from bank failure.

Following Gertler and Kiyotaki (2015), the representative household incurs a management cost $h(A_t^h)$ when directly investing in claims issued by capital-producing firms. This cost captures in a reduced-form manner the comparative disadvantage of households with respect to banks in screening and monitoring investment opportunities. It is assumed to be increasing and convex in the total direct investment in capital-producing firms by the household.

The stochastic discount factor of the household can be defined as $\Lambda_{t+1} \equiv \beta U'(C_{t+1}) U'(C_t).$

### 3.3 Bankers

Bankers are a special class of members of the household who get exclusive temporary access to the opportunity of investing their net worth as banks’ inside equity capital. Following Gertler and Kiyotaki (2010), bankers have an iid probability $1 - \varphi$ of retiring each period. When they do so, they transfer their terminal net worth to the household and are replaced by new bankers that start with an exogenous fraction $\varrho$ of the wealth managed by bankers in the previous period.

Since, as shown below, individual banks operate under constant returns to scale and bankers take returns on bank equity $R_{t+1}^E$ as given, the value function of bankers is linear in their level of net worth. The marginal value of one unit of net worth, assuming that bankers always reinvest their full amount of available wealth as bank equity, can be written as:

$$v_t = \mathbb{E}_t \left[ \Lambda_{t+1}(1 - \varphi + \varphi v_{t+1})R_{t+1}^E \right], \tag{5}$$

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8The timing convention here is that $\tilde{R}_t^D$ is the realized return on deposits after the realization of aggregate uncertainty in period $t$, while $R_{t-1}^D$ is the promised return when investment decisions are taken.

9This cost implies that, when banks’ constraints tighten and corporate claims shift to the balance sheet of households, there is an efficiency loss that translates into a higher investment cost for entrepreneurs and depressed investment.
where \((1 - \varphi + \varphi v_{t+1})\) captures the (stochastic) shadow value of net worth, which is a weighted average of the marginal values for exiting and for continuing bankers. From the expression above, it can be noted that, as long as \(v_t > 1\), it will always be optimal for the banker to reinvest its full amount of available wealth as bank equity capital.\(^{10}\) The term \(\Lambda_{t+1}^b \equiv \Lambda_t^b (1 - \varphi + \varphi v_{t+1})\) will be referred to as the bankers’ stochastic discount factor.

\[\textbf{3.3.1 Individual banks}\]

There is a continuum of measure one of perfectly competitive ex-ante identical banks. A bank lasts for one period: it is an investment project created by bankers at \(t\) and liquidated at \(t + 1\). Banks raise deposits \(D_t\) with a promised return \(R_{t+1}^D\) from households, and equity \(E_t\) from bankers. They can invest in claims issued by capital-producing firms (“corporate claims”) \(A_t^b\) and in sovereign debt \(B_t^b\). It is assumed that individual banks are not able to fully diversify their investment in capital-producing firms so that the investment \(A_t^b\) has bank-idiosyncratic returns \(\omega R_{t+1}^{K_t}\) per unit of investment, where \(\omega\) is a bank-idiosyncratic shock.\(^{11}\) The stochastic gross return of sovereign debt is \(\tilde{R}_{t+1}^B\). Banks face some net liquidity management costs \(m(D_t, B_t^b)\) which are increasing in \(D_t\) and decreasing in \(B_t^b\). Sovereign debt holdings thus help banks to reduce the cost of their deposit funding.

Banks operate under limited liability, which means that the equity payoffs generated by a bank at time \(t+1\) are given by the positive part of the difference between the returns from its assets and the repayments due to its deposits, net of the liquidity management cost \(m(D_t, B_t^b)\). If the returns from the assets are greater than the repayments and costs associated with the deposits, the difference is paid back to the bank’s equity holders. Otherwise, the bank’s equity is written down to zero and its assets are repossessed by the government, which runs the deposit insurance scheme.\(^{12}\) Each bank maximizes the net present value of

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\(^{10}\) Clearly, the household’s value of a marginal unit of wealth that bankers decided to pay back to them at \(t\) would be one.

\(^{11}\) This assumption can be interpreted as banks lending to a single firm only, or as lending to a mass of identical firms in some sector or geographical location affected by a common shock \(\omega\).

\(^{12}\) The model follows Bernanke et al. (1999) in adopting a “costly state verification” setup, by which banks’ depositors must incur a cost that is proportional to the assets of the bank in order to observe the realization.
its shareholders’ equity stakes

\[ E_t \left[ \Lambda_{t+1} \max \{ \omega R^K_{t+1} A_t^b + \tilde{R}^B_{t+1} B_t^b - R^D_t D_t - m(D_t, B_t^b), 0 \} \right] - v_t E_t, \]  

where the equity \( E_t \) is valued at its equilibrium opportunity cost \( v_t \), and the max operator reflects shareholders’ limited liability. The balance sheet identity imposes that

\[ A_t^b + B_t^b = D_t + E_t. \]  

The bank is also subject to a regulatory capital requirement

\[ E_t \geq \gamma (A_t^b + \iota B_t^b). \]  

which imposes that at least a fraction \( \gamma \) of the banks’ risk-weighted assets has to be financed with equity capital. Sovereign debt holdings \( B_t^b \) are subject to a risk weight of \( \iota \), while corporate lending \( A_t^b \) is subject to a risk weight normalized to one.

The liquidity management costs \( m(D_t, B_t^b) \) are assumed to be homogeneous of degree one, increasing in the amount of deposits \( D_t \), decreasing in the amount of the sovereign debt \( B_t^b \), and to go to infinity as \( B_t^b \) goes to zero. These costs could be justified in a model in which bank deposits were demand deposits whose holders could withdraw them at some interim period (as in Diamond and Dybvig, 1983).\(^{13}\) In such world, selling (or borrowing against) government bonds, rather than using more costly alternatives such as selling (or borrowing against) less liquid corporate claims, would allow the bank to better accommodate deposit withdrawals.\(^{14}\) Of the bank-idiosyncratic shock \( \omega \). As in Townsend (1979), this friction provides a rationale for the use of debt financing and implies a deadweight loss associated with bank failure.

\(^{13}\)In a recent paper, Bianchi and Bigio (2018) develop a microfounded dynamic model in which banks hold a precautionary buffer of liquid assets to mitigate the risk of large withdrawals of deposits.

\(^{14}\)Technically, the cost \( m(D_t, B_t^b) \) also helps to guarantee the existence of an interior solution to the bank’s portfolio problem. As shown in Repullo and Suarez (2004), one-period lived perfectly competitive banks operating under limited liability that could invest in two different risky assets would optimally specialize in one of them, unless there exist some source of complementarity between the two assets. Here the
As in Bernanke, Gertler, and Gilchrist (1999), the bank-idiosyncratic shocks $\omega$ have a unit-mean lognormal distribution. As in Christiano, Motto, and Rostagno (2014), the cross-sectional dispersion of these shocks, denoted $\sigma_t$, evolves stochastically over time, driven by some aggregate risk shocks. Those banks which draw a value of $\omega$ below the threshold

$$\bar{\omega}_{t+1} = \frac{R_t^B D_t + m(D_t, B_t^b) - \tilde{R}_{t+1}^B B_t^b}{R_{t+1}^K A_t^b},$$

will default in period $t + 1$. Section 3.7 discusses the solution to bank’s problem in detail.

### 3.3.2 Aggregation

Market clearing implies that, in equilibrium, the aggregate wealth of bankers has to be equal to the aggregate amount of equity issued by banks,

$$N_t^b = E_t.$$  \hspace{1cm} (10)

The law of motion of bankers’ aggregate level of net worth is

$$N_t^b = \varphi R_t^E E_{t-1} + (1 - \varphi) \varrho N_{t-1}^b,$$

where the first term represents retained earnings of continuing bankers, and the second term represents the initial endowment of new bankers. Transfers from retiring bankers to the household, net of the initial endowment received by new bankers, are

$$\Pi_t = (1 - \varphi) \left[ R_t^E E_{t-1} - \varrho N_{t-1}^b \right].$$

\hspace{1cm} (12)

Complementarity comes from the different degrees of liquidity of each asset. The liquidity role of public debt has been analyzed in the theoretical literature, for instance, by Woodford (1990), and Holstrom and Tirole (1998), as well as by Brutt (2011), Gennaioli et al. (2014), and Perez (2018) in the context of sovereign default models.
3.4 Government

The government issues short-term debt to finance its deficit. Its budget constraint states that, each period, the issuance of one-period debt $B_t$ has to be equal to the sum of the cost of servicing previous period debt $\tilde{R}_t^B B_{t-1}$, public spending $G_t$ minus tax revenues $T_t$, and the cost of the deposit insurance scheme $\Theta_t$:

$$B_t = \tilde{R}_t^B B_{t-1} + G_t - T_t + \Theta_t,$$

(S13)

Sovereign default events arise from the existence of a fiscal limit, which defines the maximum level of debt that the government can sustain, as in Bi (2012). As in Bi and Traum (2012) and Bocola (2016), such fiscal limit is assumed to be stochastic and to follow a logistic function that depends on the level of debt $B_t$. When such limit is exceeded, the government defaults.\(^\text{15}\) In this context, if the default event at the end of period $t$ is represented by the binary variable $\xi_{t+1} \in \{0, 1\}$, the probability of default in period $t$ is determined as

$$p_t \equiv \text{Prob}(\xi_{t+1} = 1|B_t, s_t) = \frac{\exp(\eta_1 + \eta_2 B_t + s_t)}{1 + \exp(\eta_1 + \eta_2 B_t + s_t)},$$

(S14)

where $\eta_1$ and $\eta_2$ are exogenous parameters. In addition to the level of debt $B_t$, the probability of default is driven by an exogenous variable $s_t$, that evolves stochastically and captures shocks to the default probability that are orthogonal to domestic economic conditions.\(^\text{16}\) If the government does not default ($\xi_{t+1} = 0$), it pays back the promised (gross) return $\tilde{R}_t^B$ per unit of debt to its creditors. If it defaults ($\xi_{t+1} = 1$), it writes off a fraction $\theta \in [0, 1]$ of its outstanding stock of debt and repays the remainder. Thus, the realized return of the

\(^{15}\) This specification allows to capture the positive link between the default probability and the level of debt that emerges endogenously in quantitative models of strategic default in the tradition of Eaton and Gersovitz (1981), such as Aguiar and Gopinath (2006) and Arellano (2008).

\(^{16}\) Bahaj (2019) finds that exogenous shocks orthogonal to domestic economic conditions were responsible for more than 50% of the variation in sovereign yields observed during the European debt crisis.
government bonds can be expressed as

$$\tilde{R}^B_{t+1} = (1 - \theta \xi_{t+1}) R^B_t.$$  \hfill (15)

Tax revenues, collected from households in a lump-sum fashion, are determined according to a fiscal rule

$$T_t = \tau_Y Y_t + \tau_B B_{t-1},$$  \hfill (16)

where the first term can be interpreted as the automatic-stabilizer component of tax revenues, and the second term as the debt-stabilizer component. Furthermore, government spending is assumed to be equal to a constant fraction $g$ of the steady-state level of output $\bar{Y}$,

$$G_t = g \bar{Y}.$$  \hfill (17)

### 3.4.1 Deposit insurance

Bank liabilities are partially guaranteed by the government through a deposit insurance scheme. When a bank fails, its equity capital is written down to zero. The deposit insurance scheme takes over its assets but incurs some bank resolution costs which are assumed to be a fraction $\mu$ of the assets, resulting in a deadweight loss. The proportion $1 - \xi$ of the net asset recovered is paid out to depositors in compensation for the uninsured fraction of their deposits. The insured fraction $\xi$ is fully paid out by the scheme. The resulting net liability for the government can be written as:

$$\Theta_t = \chi \left[ \left( R^D_{t-1} D_{t-1} - \tilde{R}^B_{t-1} B^h_{t-1} + m(D_t, B^h_t) \right) F_t - (1 - \mu) R^K_{t-1} A^h_{t-1} \Gamma_t \right],$$  \hfill (18)

where

$$F_t \equiv F(\bar{\omega}_t; \sigma_t) = \int_0^{\bar{\omega}_t} f(\omega; \sigma_t) d\omega, \quad \Gamma_t \equiv \Gamma(\bar{\omega}_t; \sigma_t) = \int_0^{\bar{\omega}_t} \omega f(\omega; \sigma_t) d\omega,$$  \hfill (19)

and $f(\omega; \sigma)$ is the probability density function of the idiosyncratic shocks $\omega$, conditional on the realization of the stochastic cross-sectional dispersion $\sigma$. 18
3.5 International investors

International investors are modeled as in Aguiar, Chatterjee, Cole, and Stangebye (2016). International financial markets are segmented, such that only a subset of foreign investors participates in the domestic sovereign debt market. These investors are one-period lived risk-averse agents who start with some exogenous endowment $N^*$ and are replaced by a new set of identical investors in the following period. The representative investor solves

$$\max_{B_t^*} \mathbb{E}_t U^*(C_{t+1}^*),$$

subject to the budget constraint:

$$C_{t+1}^* = \tilde{R}^B_{t+1} B_t^* + R^*(N^* - B_t^*),$$

where $B_t^*$ is the domestic sovereign debt held by foreign investors, $C_{t+1}^*$ is investors’ wealth at the end of the period, and $U^*(\cdot)$ is a standard concave, twice continuously differentiable function.\(^{17}\) International investors can invest their endowment in government bonds and they can lend (or borrow) at an international risk-free rate $R^* < 1/\beta.$\(^{18}\)

3.6 Equilibrium

A competitive equilibrium is given by the policy functions of the representative household, the representative bank, the representative firm, and the representative international investor, such that, given a sequence of equilibrium prices and a sequence of shocks, the sequence of each of the agents’ decisions solve their corresponding problems, the sequence of prices clears

\(^{17}\)Recent papers in the sovereign default literature have emphasized the role of international lenders’ risk aversion in determining sovereign risk premia (see, for example, Lizarazo, 2013; Aguiar et al., 2016; and Bianchi, Hatchondo, and Martinez, 2018). This risk aversion could capture, in a reduced form manner, balance sheet constraints faced by international investors as in Morelli, Perez, and Ottonello (2019), among other things.

\(^{18}\)This assumption, typically used in models of small open economies (see, for example, Uribe and Schmitt-Grohe, 2017), implies that, in equilibrium, the domestic economy is a net borrower from the rest of the world.
all markets, and the sequence of endogenous state variables satisfies their corresponding laws of motion. A formal definition of the competitive equilibrium, together with the complete set of optimality and market clearing conditions, is provided in Appendix B.

3.7 The problem of the bank and main mechanisms

This section discusses the solution to the problem of the bank described in equations (6) to (8), and the mechanisms underlying its risk-taking decisions. Using the notation introduced in equation (19), the objective function of the representative bank can be rewritten as:

$$\mathbb{E}_t A^{b}_{t+1} \left[ R^K_{t+1} A^{b}_t (1 - \Gamma_{t+1}) + \left( \tilde{R}^{B}_{t+1} B^{b}_t - R^D_t D_t - m(D_t, B^{b}_t) \right) (1 - F_{t+1}) \right] - v_t E_t.$$  \hspace{1cm} (22)

Combining the first order conditions with respect to the choices of $D_t$ and $E_t$ yields

$$v_t = \lambda_t + \mathbb{E}_t \left[ A^{b}_{t+1} (R^D_t + m^D_t) (1 - F_{t+1}) \right],$$  \hspace{1cm} (23)

where $\lambda_t \geq 0$ is the Lagrange multiplier associated with the regulatory capital requirement constraint (8), and

$$m^D_t \equiv \frac{\partial m(D_t, B^{b}_t)}{\partial D_t} > 0,$$

is the marginal liquidity management cost of bank deposits. Equation (23) states that, in equilibrium, the marginal cost of an additional unit of equity ($v_t$) has to be equal to the marginal benefit of relaxing the regulatory requirement constraint (8) plus the marginal cost of substituting that unit of equity with one unit of deposits. This condition implies that the capital requirement constraint will be binding ($\lambda_t > 0$) as long as

$$v_t > \mathbb{E}_t \left[ A^{b}_{t+1} (R^D_t + m^D_t) (1 - F_{t+1}) \right],$$

that is, as long as the shadow price of banker’s equity at $t$ exceeds the effective cost of deposit funding to bank shareholders (as given by the discounted value of the marginal repayments and costs incurred per unit of deposits if the bank does not fail).
The first order conditions with respect to the investments in corporate claims $A^b_t$ and sovereign bonds $B^b_t$ are, respectively,

$$
E_t \left[ \Lambda^b_{t+1} R^K_{t+1} (1 - \Gamma_{t+1}) \right] = (1 - \gamma) E_t \left[ \Lambda^b_{t+1} (R^D_{t} + m^D_t)(1 - F_{t+1}) \right] + \gamma v_t, \quad (24)
$$

$$
E_t \left[ \Lambda^b_{t+1} (\tilde{R}^B_{t+1} - m^B_t)(1 - F_{t+1}) \right] = (1 - \gamma) E_t \left[ \Lambda^b_{t+1} (R^D_{t} + m^D_t)(1 - F_{t+1}) \right] + \gamma v_t, \quad (25)
$$

which state that, in equilibrium, bankers’ marginal benefit of an additional unit of investment (in either $A^b_t$ or $B^b_t$) has to be equal to the effective weighted average cost of the funds needed to finance that investment. The marginal benefit of one additional unit of sovereign debt in (25) includes, apart from the return $\tilde{R}^B_{t+1}$, the marginal reduction in liquidity management costs

$$
m^B_t \equiv \frac{\partial m(D_t, B^b_t)}{\partial B^b_t} < 0. \quad (26)
$$

Equations (24) and (25) shed light on the main effects of higher capital requirements $\gamma$. On the one hand, a higher $\gamma$ reduces banks’ leverage. This lowers their failure risk and thus translates into lower deposit rates $R^D_t$. Cheaper deposit funding (represented by the first term on the right hand side of both equations) implies that, everything else equal, banks are willing to invest in corporate claims offering a lower yield which means that, in equilibrium, aggregate investment increases. On the other hand, a higher $\gamma$ can increase the average cost of funds for banks since, as shown above, equity is relatively more expensive than deposits. Furthermore, in equilibrium, a higher capital requirement increases the relative scarcity of bank equity, so the per-unit shadow value of equity $v_t$ also increases. More expensive equity funding (represented by the second term on the right hand side of both equations) increases the required return for banks to be willing to invest in corporate claims, which decreases aggregate investment.

Importantly, which of these two effects dominates will depend on the level of capital requirements. As it will be shown below, when leverage is high (this is, when the capital requirement $\gamma$ is low) higher capital requirements reduce bank failure risk. The subsequent
reduction in deposit rates more than compensates the increase in funding costs associated by a higher share of equity finance, leading to higher investment and economic activity. After a certain point, when leverage, bank failure risk and, therefore, deposit rates are low, by increasing the relative scarcity of bankers’ net worth, higher capital requirements make the borrowing cost of non-financial firms go up, decreasing investment and output. The quantitative results in Section 5 characterize the effects of capital regulation for the calibrated version of the model.

4 Quantitative analysis

This section outlines the computational method used to obtain the numerical solution of the model, introduces the functional forms chosen for the numerical analysis, and presents the baseline parameterization. It then explores the quantitative properties of the model and its main mechanisms.

4.1 Solution method

The model is solved using a global solution method. In particular, the method used is policy function iteration (Coleman, 1990), also known as time iteration (Judd, 1998). Functions are approximated using piecewise linear interpolation between grid points, as advocated in Richter, Throckmorton, and Walker (2014). A detailed description of the numerical solution method and a measure of its accuracy are provided in Appendices C and D, respectively.

Using global solution methods is important given the inherent non-linearities present in sovereign default models. Traditional log-linearisation methods are not able to capture the variation in risk premia (due to the certainty equivalence), which represents an important source of amplification in this model, as shown below, while higher order perturbation methods provide accurate approximations only locally, failing to capture the dynamics of models with large deviations from the steady state as the one presented here.\footnote{Aruoba, Fernandez-Villaverde, and Rubio-Ramirez (2006) and Richter, Throckmorton, and Walker (2014)} The main drawback
of using global solution methods is that they are very computationally intensive, which con-
strains the size of the models that can be feasibly solved. This is because each additional
state variable increases exponentially the size of the state space, rendering the so called curse
of dimensionality. Recent improvements in computational power and numerical solution pro-
cedures allow to solve increasingly complex models, but still pose a constraint that is not
easily overcome.\textsuperscript{20}

4.2 Functional forms and shock processes

In the quantitative analysis below, the functional form chosen for the utility function of the
household is
\begin{equation}
U(C_t) = C_{t}^{1-\nu} - \frac{1}{1 - \nu},
\end{equation}
with constant risk-aversion parameter $\nu$. Following Aguiar et al. (2016), the same functional
form and risk-aversion parameter are chosen for the utility function of international investors
$U^*(C_t^*)$. The investment management cost function, as in Gertler and Kiyotaki (2015), is
\begin{equation}
h(A_t^h) = \kappa(A_t^h)^2.
\end{equation}
The functional form for the liquidity management costs is
\begin{equation}
m(D_t, B_t^b) = \phi \left( \frac{D_t}{B_t^b} \right) D_t,
\end{equation}
which is compatible with the assumptions described in subsection 3.3.1. Bank risk shocks $\sigma_t$
evolve according to the following law of motion:
\begin{equation}
\ln \sigma_t = (1 - \rho_\omega) \ln \bar{\sigma} + \rho_\omega \ln \sigma_{t-1} + \varepsilon_t,
\end{equation}
\textsuperscript{20}For a survey, see Maliar and Maliar (2014) and Fernandez-Villaverde, Rubio-Ramirez, and Schorfheide (2016).
where $\varepsilon$ is an iid normally-distributed innovation with mean zero and standard deviation $\sigma_\omega$, while sovereign risk shocks follow

$$s_t = \rho_s s_{t-1} + \epsilon_t,$$  \hspace{1cm} (31)

where $\epsilon$ is an iid normally-distributed innovation with mean zero and standard deviation $\sigma_s$.

### 4.3 Mapping the model to the data

The model is calibrated to quarterly frequency. The calibration strategy consists of a two-step procedure. In the first step, standard parameters of the model are set to commonly agreed values in the business cycle literature, taken from related macro-banking papers, or chosen to directly match certain empirical targets observable in the data. These parameters, listed in Table 1, are mainly the ones concerning household preferences and the aggregate production function, some of the parameters in the banking side of the model, and parameters related to the fiscal part. The table summarizes the value of these parameters and their sources.

In the second step, values for the remaining parameters are set so as to jointly match a number of empirical moments using aggregate macroeconomic and financial data from Spain. Arguably, the experience in Spain during the recent financial and sovereign debt crisis provides an ideal example of the interaction between the forces and mechanisms captured by the model: an economy with strong reliance on bank funding, a government with reasonably healthy public finances before the crisis, banks with a high exposure to domestic sovereign debt, and, eventually, a severe banking crisis triggered by the end of the credit boom and the recession associated with the Global Financial Crisis.\(^{21}\) Although the calibration of these parameters is done in a joint manner, most of them can be associated to a particular empirical target, as reported in Table 1. Targeted moments consist mostly of consolidated sector financial accounts and cross-holdings of assets, as well as the cost of borrowing for

\(^{21}\) The model, however, is not expected to capture every single element of the Spanish crisis. For instance, the model is silent about the preceding credit boom linked to the construction and real estate boom started in the early 2000s.
the different sectors and other asset returns. A detailed description of all the parameters and their sources or targets is provided below. Table 2 reports the list of moments that are targeted in the calibration exercise.

**Household preferences and production function.** The subjective discount rate $\beta$ and the risk-aversion parameter $\nu$ of the representative household are set equal to standard values in the literature of 0.99 and 2, respectively. Similarly, the elasticity of physical capital $\alpha$ and its depreciation rate $\delta$ are set to 0.33 and 0.025, respectively.

**Banking sector.** The capital requirement $\gamma$ is set to 8% of risk-weighted assets, consistent with the general requirement for banks under the Basel II regulatory framework (BCBS, 2004; part 2.I, paragraph 40). The risk weight assigned to domestic sovereign exposures $\iota$ is set to zero (BCBS, 2004; part 2.II, paragraph 54). The bank bankruptcy cost (the fraction of the banks’ asset value that cannot be recovered in case of bankruptcy) is set to 0.3, as in Mendicino et al. (2018).

The investment management cost parameter $\kappa$ is equal to 0.0003. It targets the share of bank finance relative to total external finance of non-financial corporations.\(^{22}\)

Two parameters drive the scarcity of bank equity in equilibrium. Intuitively, this means that they directly affect the excess return of assets intermediated by banks. These are the bankers’ net worth retention rate $\varphi$ (the complement of the bankers’ exit rate), and the parameter $\varrho$ determining the endowment of new bankers. They are set to 0.975 and 0.01, respectively, so that (i) the average spread between the rate return on corporate claims and the risk-free rate ($R^k - R^*$) matches the average spread of corporate debt; and (ii) the average return on bank equity matches its data counterpart. The liquidity management cost parameter $\phi$ is set to $1.5 \cdot 10^{-5}$. It allows to match the average exposure to sovereign debt as a fraction of bank assets.

The parameter $\sigma$, which determines the average cross-sectional dispersion of idiosyncratic

\(^{22}\)Similarly to De Fiore and Uhlig (2011) and Mendicino et al. (2018), this is defined as the share of total liabilities in the consolidated balance sheet of the non-financial corporate sector that is held by domestic banks. See Appendix A for further details.
### Table 1: Baseline parameterization

<table>
<thead>
<tr>
<th>Externally-calibrated parameters</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Household preferences</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$ Subjective discount rate</td>
<td>0.99</td>
<td>Standard</td>
</tr>
<tr>
<td>$\nu$ Risk aversion</td>
<td>2</td>
<td>Standard</td>
</tr>
<tr>
<td><strong>Aggregate production function</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$ Output elasticity of capital</td>
<td>0.33</td>
<td>Standard</td>
</tr>
<tr>
<td>$\delta$ Depreciation rate of capital</td>
<td>0.025</td>
<td>Standard</td>
</tr>
<tr>
<td><strong>Banking sector</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$ Capital requirement</td>
<td>0.08</td>
<td>BCBS (2004)</td>
</tr>
<tr>
<td>$\iota$ Risk weight of sov. bonds</td>
<td>0.0</td>
<td>BCBS (2004)</td>
</tr>
<tr>
<td>$\mu$ Bankruptcy cost</td>
<td>0.30</td>
<td>Mendicino et al. (2018)</td>
</tr>
<tr>
<td><strong>Government</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$ Write-off parameter</td>
<td>0.55</td>
<td>Zettelmeyer et al. (2013)</td>
</tr>
<tr>
<td>$\chi$ Fraction of insured deposits</td>
<td>0.46</td>
<td>Demirgüç-Kunt et al. (2015)</td>
</tr>
<tr>
<td>Internally-calibrated parameters</td>
<td>Value</td>
<td>Target</td>
</tr>
<tr>
<td><strong>Banking sector</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa$ Investment management cost</td>
<td>0.0003</td>
<td>Bank to non-bank finance</td>
</tr>
<tr>
<td>$\varphi$ Earnings retention rate</td>
<td>0.975</td>
<td>Average return on bank equity</td>
</tr>
<tr>
<td>$\phi$ Liquidity management cost</td>
<td>$1.5 \cdot 10^{-5}$</td>
<td>Sov. exposures to bank assets</td>
</tr>
<tr>
<td>$\rho$ Initial endowment</td>
<td>0.01</td>
<td>Average corporate spread</td>
</tr>
<tr>
<td>$\sigma$ Avg. dispersion of iid shocks</td>
<td>0.025</td>
<td>Average bank spread</td>
</tr>
<tr>
<td>$\sigma_{\omega}$ Volatility bank risk shock</td>
<td>0.17</td>
<td>Volatility bank spread</td>
</tr>
<tr>
<td><strong>Government</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g$ Government spending</td>
<td>0.18</td>
<td>Govt. spending to output</td>
</tr>
<tr>
<td>$\tau_Y$ Tax revenue sensitivity to income</td>
<td>0.12</td>
<td>Tax revenue to output</td>
</tr>
<tr>
<td>$\tau_B$ Tax revenue sensitivity to debt</td>
<td>0.05</td>
<td>Govt. debt to output</td>
</tr>
<tr>
<td>$\eta_1$ Sovereign risk intercept</td>
<td>-16</td>
<td>Average sovereign spread</td>
</tr>
<tr>
<td>$\eta_2$ Sovereign risk sensitivity to debt</td>
<td>1.2</td>
<td>Correlation sov. spread and debt</td>
</tr>
<tr>
<td>$\sigma_s$ Volatility sov. risk shocks</td>
<td>0.7</td>
<td>Volatility sovereign spread</td>
</tr>
<tr>
<td><strong>International investors</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^*$ Risk-free rate</td>
<td>1.008</td>
<td>German bond yield</td>
</tr>
<tr>
<td>$N^*$ Investors endowment</td>
<td>3</td>
<td>Share of debt held abroad</td>
</tr>
</tbody>
</table>
Table 2: Calibration targets and model fit

<table>
<thead>
<tr>
<th>Target</th>
<th>Description</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[G/Y]$</td>
<td>Govt. spending to GDP (%)</td>
<td>17.94</td>
<td>17.28</td>
</tr>
<tr>
<td>$E[T/Y]$</td>
<td>Tax revenue to GDP (%)</td>
<td>19.09</td>
<td>20.43</td>
</tr>
<tr>
<td>$E[B/Y]$</td>
<td>Sovereign debt to GDP (%)</td>
<td>41.18</td>
<td>44.36</td>
</tr>
<tr>
<td>$E[B^*/B]$</td>
<td>Share of debt held by non-residents (%)</td>
<td>65.43</td>
<td>61.32</td>
</tr>
<tr>
<td>$E[B^b/(A^b + B^b)]$</td>
<td>Sovereign exposures to bank assets (%)</td>
<td>7.88</td>
<td>7.93</td>
</tr>
<tr>
<td>$E[A^b/K]$</td>
<td>Share of capital financed by banks (%)</td>
<td>88.77</td>
<td>88.22</td>
</tr>
<tr>
<td>$E[R^r]$</td>
<td>International risk-free rate (%)</td>
<td>3.25</td>
<td>3.25</td>
</tr>
<tr>
<td>$E[R^E]$</td>
<td>Average return on bank equity (%)</td>
<td>10.15</td>
<td>9.68</td>
</tr>
<tr>
<td>$E[R^K - R^r]$</td>
<td>Average corporate spread (pp)</td>
<td>1.34</td>
<td>1.51</td>
</tr>
<tr>
<td>$E[R^D - R^r]$</td>
<td>Average bank spread (pp)</td>
<td>0.72</td>
<td>0.84</td>
</tr>
<tr>
<td>$E[R^S - R^r]$</td>
<td>Average sovereign spread (pp)</td>
<td>0.32</td>
<td>0.20</td>
</tr>
<tr>
<td>std($R^D - R^r$)</td>
<td>Volatility bank spread (pp)</td>
<td>0.81</td>
<td>0.73</td>
</tr>
<tr>
<td>std($R^S - R^r$)</td>
<td>Volatility sov. spread (pp)</td>
<td>1.01</td>
<td>1.17</td>
</tr>
<tr>
<td>cor($R^S, B$)</td>
<td>Correlation sov. spread and debt level (%)</td>
<td>81.61</td>
<td>75.46</td>
</tr>
</tbody>
</table>

Notes: Model column reports values at the stochastic steady state except for the last two rows, which refer to moments of the ergodic distribution of the model. Data column reports statistics calculated over the period 1999Q1-2009Q4, except for the last two row, where the standard deviation and the correlation reported are for the period 1999Q1-2014Q4. Asset returns are reported in annualized rates.

shocks $\omega$, is equal to 0.025. The volatility of shocks to this cross-sectional dispersion $\sigma_\omega$ is set to 0.17. As key determinants of the riskiness of bank returns, these two parameters allow to match the average and the standard deviation of bank deposit spreads.

**Government.** The write-off parameter for sovereign debt $\theta$ is set to 0.55, which is the value that Zettelmeyer, Trebesch, and Gulati (2013) report for the case of the Greek debt restructuring in 2012. Following Mendicino et al. (2018), the share $\chi$ of insured deposits is taken from Demirgüç-Kunt et al. (2015), who report that 46% of deposits in Spain are

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23This value is also close to the loss given default of 45% assigned to sovereign exposures under the foundation approach of Basel II (BCBS, 2004; part 2.III, paragraph 287).
covered by deposit insurance.

The level of government spending as a fraction of steady state output \( g \) is set to 0.18. The parameters \( \tau_Y \) and \( \tau_B \) governing the fiscal rule are set to 0.12 and 0.05, respectively, targeting average tax revenues and the ratio of debt-to-GDP.\(^{24}\)

The parameters of the fiscal limit distribution, which determine the probability of default as a function of the outstanding level of debt, target the average sovereign yield spread with respect to the risk-free rate, and the correlation between the level of debt and sovereign spreads. The volatility \( \sigma_s \) of exogenous sovereign risk shocks targets the standard deviation of sovereign yields.

**International investors.** The international risk-free rate \( R^* \) is equal to 1.008, which matches the annualized yield of one-year German bonds. The endowment \( N^* \), which matches the share of domestic sovereign debt held abroad, is set to 3.

### 4.4 Sovereign-bank nexus dynamics

In this section, simulations of the model are used to evaluate the ability of the model to account for the actual dynamics of the sovereign-bank nexus observed during the European crisis. To this end, the model is simulated for 200,000 periods and the simulated time series are used to construct eight-year event windows centered around the peak of crisis episodes. A crisis episode is defined as a situation in which both the sovereign spread and bank spreads are at least two standard deviations above their unconditional mean. The period \( t \) around which these events are centered is selected as the one in which sovereign spreads reach their peak. Figure 3 depicts the median event, as well as the 25th and the 75th percentile events.\(^{25}\) The depicted variables include sovereign, bank, and corporate spreads, sovereign debt (as a percentage of output), banks’ exposure to sovereign debt (as a percentage of total assets),

\(^{24}\)Given that the model abstracts from bonds held by domestic agents other than banks, the stock of debt measured in the data is that in the balance sheet of domestic monetary financial institutions (other than the central bank) and debt held by non-residents.

\(^{25}\)In order to make the simulations of the model comparable with the data, crisis events are selected only among those in which sovereign default does not materialize.
Figure 3: Sovereign-bank nexus event windows: data and model simulations

Notes: Event windows selected for periods with above-normal sovereign and bank spreads. Each window represents the median event (solid blue lines) identified in a 200,000-period simulation. Thin blue lines depict the 25th and 75th percentiles. Orange lines depict data counterparts for Spain from 2008Q3 to 2016Q3.

the share of total outstanding debt held by foreign investors, and the deviation from their HP trend of output, consumption, and bank credit to the non-financial corporate sector (which is taken as the data counterpart of the corporate claims in which banks invest in the model).

Data counterparts are selected by identifying the quarter in recent Spanish data in which sovereign spreads peaked, and taking an eight-year window around it. This quarter is 2012Q3, so the depicted window corresponds to the period 2008Q3-2016Q3.

Figure 3 shows that the model replicates reasonably well some of the key features of the
sovereign debt crisis in Spain. In particular, the simulations of the model match closely the magnitude of the increase in borrowing costs for the government, banks and non-financial companies until the peak in 2012. After they peak, however, model spreads remain higher for longer than their data counterparts. One plausible explanation for the discrepancy is that the model abstracts from the unconventional monetary policy measures which were announced by the European Central Bank during the summer of 2012.26

The increase in sovereign debt is also captured. In the model, such increase is the result of the surge in the cost for the deposit insurance scheme, the weaker economic conditions and lower tax revenues, and the rise in the cost of debt financing. In the data, however, government debt starts from below the median levels and grows faster to levels such as those predicted by the model. This possibly reflects the boom and bust elements of the Spanish real estate crisis which the model does not specifically capture. Sovereign bond holdings shift from international investors to domestic banks, increasing the overall failure risk faced by the latter. In quantitative terms, the model overestimates the increase in exposures of domestic banks. One reason for this is that, the model abstracts from assets in the balance sheet of banks other than bank credit to non-financial borrowers and sovereign exposures (such as mortgages) that, due to their longer maturity, did not contract as much during this period.

In terms of other macroeconomic outcomes, the model captures pretty well both the magnitude and the time profile of the contraction in output, consumption, and bank credit to private non-financial borrowers during the crisis. Overall, these results document the ability of the model to capture the effects of the sovereign-bank nexus on financial stability and macroeconomic activity.

26 Afonso et al. (2018) provides evidence of a regime switch in sovereign bond prices following the announcement of the Outright Monetary Transactions (OMT) programme in August 2012, which weakened the link between spreads and fundamentals.
5 Counterfactual exercises

This section presents two counterfactual exercises. First, it assesses the contribution of sovereign risk as an amplification mechanism. To do so, it compares crises in the baseline model with those obtained from a parameterization in which sovereign debt is always risk free. Second, it evaluates the effects of capital regulation.

5.1 The model without sovereign risk

The model with risk-free sovereign debt allows to gauge the amplification effect of sovereign risk during crises. Figure 4 depicts the median crises in the model without sovereign risk when the simulations of the model are fed the same sequence of shocks as in Figure 3.

In this counterfactual scenario, even though the sequence of shocks is identical to that in the baseline economy of Figure 3, sovereign default risk and, therefore sovereign yield spreads remain constant at zero. While banks in the baseline model react to the spike in interest rates paid by government bonds by substantially increasing their exposure to sovereign debt, their debt holdings remain low and relatively constant when default risk is shut down.

When sovereign risk is present, the increase in banks’ sovereign debt holdings drive up bank failure risk, which translates into higher funding costs. This has a large impact on banks’ profitability, and the abnormally low returns further reduce the aggregate level of bank equity over time, with an associated decrease in bank lending. When sovereign risk is shut down, bank funding costs do not grow as much, allowing bank lending to not fall as much and to recover relatively quicker.

These results illustrate the amplification effects that sovereign default risk has on the banking sector and, through them, on the funding conditions faced by the corporate sector. As shown in Figure 4, an initial sequence of shocks to idiosyncratic bank risk translate into system-wide instability through the endogenous contagion effect that the increase in sovereign risk has on the failure risk of banks. Importantly, these effects are sizeable even if the default
Figure 4: Event windows: risk-free sovereign debt counterfactual

Notes: Event windows selected for periods with above-normal sovereign and bank spreads in the baseline model. Each window represents the median event (solid blue lines) identified in a 200,000-period simulation. Dotted red lines depict simulations under the same sequence of shocks in the counterfactual model without sovereign risk.

The increase in banks’ funding costs and the resulting decrease in their profitability, in addition to the high yield paid by sovereign bonds, encourages banks to increase their exposure to sovereign risk. Because of the mispricing of risk at the margin, individual banks

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27 This narrative is consistent with the situation faced in countries such as Spain and Ireland in the aftermath of the Global Financial Crisis.
do not internalize the effect of their increased riskiness on the funding costs of the whole banking sector. Furthermore, because of limited liability, they can enjoy the high returns from holding sovereign bonds as long as the government does not default, while suffering losses limited to their initial equity contribution in case the default materializes, effectively shifting the risk of failure to their depositors. Thus, the results seem to point to a macroprudential rationale for regulation making banks internalize the effects of their sovereign exposures and in mitigating the negative effects of the feedback loop.

5.2 The role of bank capital requirements

This section analyzes the implications of bank capital regulation for the sovereign-bank nexus. Figure 5 presents crisis dynamics under the same sequence of shocks for a number of parameterizations where the risk weight \( \iota \) applied to banks’ sovereign bond holdings in the calculation of regulatory capital requirements is increased from its initial level of zero. Each blue line depicts the median trajectories under a different risk weight \( \iota \), each representing an additional 10 pp risk weight, with lighter colors representing higher values, from 10% to 100%.

As discussed in Section 3.7, increasing capital requirements for banks’ sovereign exposures has two effects. First, for the same yield, it makes investing in sovereign debt less attractive. This is because the cost of equity is higher than the cost of deposits. Second, the funding coming from bankers (their ‘skin-in-the-game’) increases, reducing the risk-taking incentives associated with high leverage. This translates into lower funding costs, less amplification effects and quicker recoveries from the initial shock. Each increase in the risk weight \( \iota \) decreases funding costs for both banks and private non-financial borrowers, suggesting that positive risk weights are effective in mitigating the negative effects of the increase in sovereign risk on financial stability.

However, the benefits of increasing the risk weight for sovereign exposures do not come at no cost. One of the most important effects is that initial contractions in lending become sharper at the beginning of crises. This is because banks are now required to use part of their limited amount of equity to back their sovereign bond holdings, which leaves them
Figure 5: Event windows: The role of positive risk weights

Notes: Event windows selected for periods with above-normal sovereign and bank spreads in the baseline model. Each window represents the median event (solid black lines) identified in a 200,000-period simulation. Solid blue lines depict the simulations of counterfactual economies with higher risk weights $\iota$ for sovereign exposures (lighter shades of blue represent higher risk weights).
Figure 6: Event windows: The role of capital requirements

Notes: Event windows selected for periods with above-normal sovereign and bank spreads in the baseline model. Each window represents the median event (solid black lines) identified in a 200,000-period simulation. Solid blue lines depict the simulations of counterfactual economies with higher capital requirements $\gamma$ (lighter shades of blue represent higher capital requirements).
with a lower amount of equity available for other purposes, effectively crowding out banks’ corporate lending. Thus, the drop in banks’ investment when equity is relatively more scarce is amplified. Nevertheless, lending recovers quicker than in the baseline case with zero risk weights due to the overall decrease in bank risk and the subsequent quicker recovery of aggregate bank capital.

Alternatively, the model also allows to assess the effect of an increase in the general capital requirement $\gamma$. Figure 6 presents crisis dynamics under the same sequence of shocks under a number of parameterizations where the capital requirement $\gamma$ is increased from its initial level of 8%. Blue lines depict the impulse-response function under a different capital requirement $\gamma$, each representing a 10 pp increment, with lighter colors representing higher values, from 9% to 20%.

As in the case with higher risk weights, higher capital requirements reduce the failure risk of banks by reducing their leverage. This, in turn, translates into lower borrowing costs, which lead to a higher recovery of bank equity. In contrast to positive risk weights, however, higher capital requirements do not have the undesirable effect of crowding out bank lending at the beginning of crises. This reduces corporate spreads and leads to a quicker recovery in economic activity. Additionally, higher capital requirements, by reducing bank leverage and making banks safer, also make crises less frequent ex-ante, which also helps mitigating the negative consequences of the sovereign-bank nexus.

5.3 Social welfare and optimal capital requirements

It is possible to compare social welfare under different values of capital requirements $\gamma$ and of risk weights $\iota$. Social welfare is computed as the expected value of the household intertemporal utility, calculated by averaging across a large number of simulations of the model economy. More formally, the proposed measure of welfare $W_0(\gamma, \iota)$, as a function of the capital requirement $\gamma$ and the risk weight $\iota$, can be defined as

$$W_0(\gamma, \iota) = E_0 \left[ \sum_{t=0}^{\infty} \beta^t U(C_t; \gamma, \iota) \right].$$

(32)
Welfare can be expressed in terms of equivalent permanent consumption units by obtaining the value $\bar{C}(\gamma, \iota)$ that solves
\[
W_0(\gamma, \iota) = \frac{U(\bar{C}(\gamma, \iota))}{1 - \beta}.
\] (33)

Figure 7 illustrates the effect of capital regulation on social welfare, measured as equivalent permanent consumption units. The right panel shows that, for the baseline capital requirement $\gamma = 8\%$, risk weights higher than zero are welfare improving. The reasons are intuitive, provided the effects that higher risk weights have on banks’ endogenous exposure to sovereign risk, as described above. This result, however, may also come from the positive effects of a general reduction in bank leverage, that can also be achieved by increasing the general capital requirement $\gamma$, instead of specifically targeting sovereign debt exposures.

The left panel shows that the optimal capital requirement $\gamma$ sits above the baseline level of 8%. Increases in the capital requirement from the initial level generate large gains in terms of social welfare, reaching a unique welfare-maximizing interior point at 14%. Interestingly,

![Figure 7: Social welfare as a function of capital regulation](image_url)

Notes: The left panel depicts social welfare, measured in equivalent permanent consumption units, as a function of the capital requirement $\gamma$, when the risk weight $\iota$ is fixed at its baseline value of zero. The right panel depicts the percentage change in social welfare as a function of the risk weight $\iota$, with respect to the zero risk weights scenario, for two different values of $\gamma$: its baseline value of 8% and its optimal value $\gamma^* = 14\%$. 

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the optimal capital regulation is attained when the risk weight $i$ is at zero, provided that the capital requirement $\gamma$ is at its optimal level of 14%.

Figure 8 depicts the change in the average value of some of the main endogenous macroeconomic and financial variables of the model economy for different values of the capital requirement $\gamma$.\textsuperscript{28} It shows the negative relationship between capital requirements and bank spreads, which are reduced as bank leverage and, therefore, bank failure risk decrease. It also illustrates how, for capital requirements below 10%, the reduction in deposit rates translates into a lower average cost of funds, which allow to reduce corporate borrowing costs and increase bank intermediation and output. As anticipated in Section 3.7, after bank failure risk and deposit spreads become sufficiently low, the relative scarcity of equity that results from higher capital requirements start to dominate, increasing the average cost of funds for banks. This has the effect of increasing borrowing costs for non-financial firms, which decrease their investment and lower the aggregate level of output. It is also noticeable the effect on the level of sovereign debt, which decreases as a result of the lower costs of government guarantees. Interestingly, as shown in the bottom right panel, the optimal capital requirement is not the one that maximizes bank lending to non-financial borrowers. This highlights the existence of non-trivial welfare trade offs when choosing the optimal value of $\gamma$, in terms of the level of investment versus the safety of the banking sector.

Figure 9 depicts the change in the average value of the same endogenous variables, now as a function of the risk weight $i$, for two different levels of the capital requirement $\gamma$ (the baseline level of 8%, represented by solid blue lines, and the optimal level of 14%, represented by dashed red lines). When bank leverage is high (in the case of the baseline capital requirement $\gamma = 8\%$) increases in the risk weight for sovereign exposures, by reducing banks’ incentives to invest in sovereign debt, have a noticeable impact on bank deposit spreads. As a result, corporate borrowing costs become lower, stimulating investment and output. On the contrary, when the capital requirement is set to its optimal level ($\gamma = 14\%$) and bank leverage is low, increases in the risk weight $i$ have negligible effects on the cost of deposits.

\textsuperscript{28}These averages are calculated as the mean of the ergodic distribution of the model.
Figure 8: Changes in endogenous variables as a function of capital requirements

Notes: Horizontal axes represent the value of the capital requirement $\gamma$ (in percentage points), going from 8% to 24%. Each panel represents the change in one of the model’s endogenous variables, relative to the parameterization with $\gamma = 8%$. 
Figure 9: Changes in endogenous variables as a function of sovereign risk weights

Notes: Horizontal axes represent the value of the risk weight for sovereign exposures $\iota$ (in percentage points), going from 0% to 100%. Each panel represents the change in one of the model’s endogenous variables, relative to the parameterization with $\iota = 0$. 

Baseline capital requirement ($\gamma = 8\%$) - Optimal capital requirement ($\gamma = 14\%$)
When this happens, the effect on the scarcity of bank equity dominates, increasing the cost of funds for non-financial firms, with contractionary effects on investment and output.

6 Concluding remarks

This paper examines the nexus between sovereign and banking crises, and the potential effects of bank capital regulation in mitigating it by discouraging banks’ endogenous exposure to sovereign risk. To this purpose, it develops a dynamic general equilibrium model in which banks decide on their exposure to sovereign debt issued by a government subject to default risk.

One of the contributions of the model presented in this paper is that it features both endogenous bank failure risk and sovereign default risk, which have reinforcing effects on each other (what has been called the negative feedback loop between banks and sovereigns). The model allows to study the macroeconomic consequences of such feedback effects: the impact of an increase in bank failure on the probability of a sovereign default resulting from government guarantees, the endogenous increase in banks’ exposure to sovereign risk, and the feedback effects that an increase in the sovereign default risk have on banks’ solvency and their funding costs. In this sense, the possibility of a sovereign default acts as an important source of systemic risk, by which an initial shock to a small fraction of banks can translate into system-wide instability.

Distortions resulting from banks’ limited liability make investing in risky sovereign debt attractive for banks, who enjoy high profits insofar as the government does not default and suffer losses limited to their initial equity contributions otherwise. These risk-shifting incentives result in excessive exposure to sovereign risk. Higher bank risk translates into higher funding costs. When banks do not internalize the effect of their individual risk-taking choices on the funding costs of the whole banking system, sovereign exposures might pose an externality that can justify the use of bank capital regulation.

By disrupting banks’ intermediation ability, the effects of the feedback loop have dramatic consequences for economic activity, even when the sovereign default event does not materialize.
ex-post. Thus, the model environment provides a rationale for macroprudential policies aimed to reduce banks’ risk-shifting incentives.

The model is used to address some of the central issues in recent discussions about the current regulatory treatment of banks’ exposure to (domestic) sovereign debt. In particular, the paper analyzes the potential macroprudential role of capital requirements for sovereign debt. The main finding is that a positive risk weight for sovereign debt holdings in the calculation of capital requirements both reduces banks’ endogenous exposure to sovereign risk and makes bank effectively safer and, consequently, helps mitigating the two-way feedback effects between banking and sovereign crises and its negative spillovers on economic activity. This is particularly the case when capital requirements are relatively low, and thus bank leverage is high. However, the optimal regulation prescribes a relatively high level of capital requirements (higher than in the baseline calibration of the model) together with zero risk weights for sovereign exposures.

Other sets of macroprudential policies could also be analyzed in the context of the model, such as time-varying capital requirements, and concentration limits to the exposure of banks to domestic sovereign debt, among others.

The model, for the most part, abstracts from the international dimension of the sovereign-bank nexus. Understanding the effects of international spillovers would be relevant in the context of a monetary union and could shed light on issues such as common deposit insurance mechanisms and common resolution regimes. These appear to be interesting topics for a future research agenda.
References


Appendix

A Data sources


Bank to non-bank finance: Financial balance sheet of non-financial corporations (Table 2.34a), Financial Accounts of the Spanish Economy. Total liabilities are constructed as the sum of debt securities (AF.3), loans (AF.4), and other liabilities (AF.7/8), net of the share held by other non-financial corporations (S.11), general government (S.13), and the rest of the world (S.2). From those, the fraction held by monetary financial institutions (S.121/3) is calculated. Available at https://www.bde.es/webbde/en/estadis/infoest/temas/sb_cfesp.html.

Bank spreads: Time series constructed by combining two different sources. First, bank debt spreads are obtained from the dataset constructed in Gilchrist and Mojon (2018), which uses the procedure in Gilchrist and Zakrakov (2012) to calculate the average spreads on the yield of Euro area private sector bonds relative to the yield on German federal government securities of matched maturities. Monthly updates are available at https://publications.banque-france.fr/en/economic-and-financial-publications-working-papers/credit-risk-euro-area. Second, deposit interest rates are obtained from the European Central Bank Statistical Data Warehouse. In particular, data on deposits from households with agreed maturity up to one year (new business) are used (MIR.M.ES.B.L22.F.R.A.2250.EUR.N ). Available at https://sdw.ecb.europa.eu/quickview.do?SERIES_KEY=124.MIR.M.ES.B.L22.F.R.A.2250.EUR.N. Spreads are calculated relative to the yield on German 1Y bond (obtained from Bloomberg). A weighted average between bond and deposit spreads is calculated using the fraction of insured deposits $\chi$ in the baseline calibration of the model described in Section 4.3.

Sovereign spreads: Difference between the yield on a 10Y bond and the yield on a German sovereign bond with the same maturity, obtained from Bloomberg.


Bank equity returns: obtained from the Bank of Spain financial institution statistics (statistics based on individual supervisory statements, available at https://www.bde.es/webbde/en/estadis/infoest/temas/sb_ifisup.html). Calculated as total after-tax bank book profit (Table 4.36) divided by total equity (Table 4.7).
B Equilibrium equations

This Appendix presents the complete set of equilibrium equations and provides the formal
definition of a competitive equilibrium.

B.1 Households

The problem of the representative household (2) results in the following optimality conditions:

\[ \mathbb{E}_t \left[ \Lambda_{t+1} \tilde{R}_{t+1}^D \right] = 1, \tag{B.1} \]
\[ \mathbb{E}_t \left[ \Lambda_{t+1} R_{t+1}^K \right] = 1 + h'(A_t^h). \tag{B.2} \]

The household’s budget constraint is given by

\[ C_t + D_t + A_t^h + h(A_t^h) = W_t + \tilde{R}_t^D D_{t-1} + R_t^K A_{t-1}^h + \Pi_t - T_t, \tag{B.3} \]

and level of the household’s net worth \( N_t^h \) evolves according to the following law of motion:

\[ N_t^h = W_t + \tilde{R}_t^D D_{t-1} + R_t^K A_{t-1}^h + \Pi_t - T_t. \tag{B.4} \]

The stochastic discount factor of the household can be defined as

\[ \Lambda_{t+1} \equiv \beta \frac{U''(C_{t+1})}{U'(C_t)}, \]

and the realized (gross) return on deposits is

\[ \tilde{R}_t^D \equiv R_{t-1}^D - (1 - \chi) \Psi_t, \]
B.2 Bankers

The level of bankers' net worth $E_t$ evolves according to the following law of motion:

$$N_t^b = \varphi R_t^E E_{t-1} + (1 - \varphi) \rho N_{t-1}^b. \quad \text{(B.5)}$$

The marginal value of one unit of net worth for the bankers is

$$v_t = \mathbb{E}_t \left[ \Lambda_{t+1} (1 - \varphi + \varphi v_{t+1}) R_{t+1}^E \right].$$

The stochastic discount factor of the banker can be defined as

$$\Lambda_{t+1}^b = \Lambda_{t+1} (1 - \varphi + \varphi v_{t+1}).$$

B.3 Banks

The problem of the representative bank (6) results in the following optimality conditions:

$$\mathbb{E}_t \left[ \Lambda_{t+1}^b \left( R_{t+1}^K (1 - \Gamma_{t+1}) - (1 - \gamma) (R_{t+1}^D + m_t^D) (1 - F_{t+1}) \right) \right] = \gamma v_t, \quad \text{(B.6)}$$

$$\mathbb{E}_t \left[ \Lambda_{t+1}^b \left( \tilde{R}_{t+1}^B + m_t^B - (1 - \gamma) (R_{t+1}^D + m_t^D) (1 - F_{t+1}) \right) \right] = \gamma \nu v_t, \quad \text{(B.7)}$$

where

$$m_t^D \equiv \frac{\partial m(D_t, B_t^b)}{\partial D_t}, \quad m_t^B \equiv \frac{\partial m(D_t, B_t^b)}{\partial B_t^b}$$

are the derivatives of the liquidity management cost with respect to deposits and sovereign bonds, respectively, and

$$\Gamma_t \equiv \Gamma(\tilde{\omega}_t; \sigma_t) = \int_{\tilde{\omega}_t} \omega f(\omega; \sigma_t) \, d\omega = \Phi \left( \frac{\log(\tilde{\omega}_t) - \sigma_t^2/2}{\sigma_t} \right), \quad \text{(B.8)}$$

$$F_t \equiv F(\tilde{\omega}_t; \sigma_t) = \int_{\tilde{\omega}_t} f(\omega; \sigma_t) \, d\omega = \Phi \left( \frac{\log(\tilde{\omega}_t) + \sigma_t^2/2}{\sigma_t} \right), \quad \text{(B.9)}$$
where \( f(\omega; \sigma) \) is the probability density function of the idiosyncratic shock \( \omega \), conditional on the volatility parameter \( \sigma \), and \( \Phi(\cdot) \) is the cumulative distribution function of the standard normal.

The balance sheet constraint is given by

\[
A_t^b + B_t^b = D_t + E_t, \tag{B.10}
\]

and the regulatory capital requirement imposes that

\[
E_t = \gamma(A_t^b + \iota B_t^b). \tag{B.11}
\]

### B.4 Producers

The problem of the representative final good producer results in the following optimality conditions:

\[
r_t^K = \alpha K_t^{\alpha - 1} L_t^1 - \alpha, \tag{B.12}
\]

\[
W_t = (1 - \alpha) K_t^{\alpha} L_t^{-\alpha}. \tag{B.13}
\]

### B.5 Government

The level of government debt outstanding \( B_t \) evolves according to the following law of motion:

\[
B_t = (1 - \theta \xi_t) R_{t-1}^B B_{t-1} + G_t - T_t + \Theta_t. \tag{B.14}
\]
Deposit insurance liabilities and the loss for depositors due to bank failure can be expressed, respectively, as

\[ \Theta_t = \chi \left( R_t^D D_{t-1} + m_t - \tilde{R}_t^B B_{t-1}^b \right) F_t - (1 - \mu) R_t^K A_{t-1}^b \Gamma_t \], \quad (B.15) 

\[ \Psi_t D_{t-1} = \left( R_{t-1}^D D_{t-1} + m_t - \tilde{R}_t^B B_{t-1}^b \right) F_t - (1 - \mu) R_t^K A_{t-1}^b \Gamma_t \]. \quad (B.16)

### B.6 International investors

The problem of the representative international investor (20) results in the following optimality condition:

\[ \mathbb{E}_t \left[ (\tilde{R}_{t+1}^B - R) U^{*'} \left( \tilde{R}_{t+1}^B B_t^* + R (N^* - B_t^*) \right) \right] = 0. \] \quad (B.17)

### B.7 Market clearing

Every period, the aggregate level of bankers’ net worth must equal the bank equity issued by banks:

\[ E_t = N_t^b, \] \quad (B.18)

the supply of government bonds must equal the bonds held by the banks and the international investors:

\[ B_t = B_t^b + B_t^*, \] \quad (B.19)

claims issued by entrepreneurs to finance investment in physical capital must equal claims held by households and banks:

\[ A_t = A_t^h + A_t^b, \] \quad (B.20)

physical capital rented by final good producers must equal the stock of capital produced by entrepreneurs:

\[ K_t = A_{t-1}, \] \quad (B.21)
and labor hired by the firm must equal the unit of labor inelastically supplied by the household:

\[ L_t = 1. \]  
\[(B.22)\]

B.8 Equilibrium

In equilibrium, the state of the economy at any date \( t \) can be summarized by three state variables collected in the vector \( S = \{ N^h_t, N^b_t, B_t \} \): aggregate net worth of the representative household \( N^h_t \), aggregate net worth available to active bankers \( N^b_t \), and the level of sovereign debt outstanding \( B_t \). Formally:

**Definition 1** A competitive equilibrium is given by the policy functions for the representative bank \( A^b_t(S), B^b_t(S), D_t(S), E_t(S) \), the representative household \( C_t(S), D_t(S), A^h_t(S) \), the representative firm \( K_t(S), L_t(S) \), and the representative international investor \( B^*_t(S) \), which determine the actions of each of the agents for each triple \( S = \{ N^h_t, N^b_t, B_t \} \), such that, given prices \( v_t(S), R^D_t(S), R^B_t(S), r^K_t(S), W_t(S) \) and the realization of the shocks:

1. The sequence of consumption and saving decisions \( \{ C_t, D_t, A^h_t \}_{t=0,1,...} \) solves the problem of the representative household, ie equations (B.1) to (B.3).

2. The sequence of portfolio choices \( \{ A^B_t, B^B_t \}_{t=0,1,...} \) and liability structure \( \{ D_t, E_t \}_{t=0,1,...} \) solves the problem of the representative bank, ie equations (B.6) to (B.11).

3. The sequence of input choices \( \{ K_t, L_t \}_{t=0,1,...} \) solves the problem of the representative firm, ie equations (B.12) and (B.13).

4. The sequence of portfolio choices \( \{ B^*_t \}_{t=0,1,...} \) solves the problem of the representative international investor, ie equation (B.17).

5. The sequence of prices \( \{ v_t, R^D_t, R^K_t, R^B_t, W_t \}_{t=0,1,...} \) clears the equity market, the deposits market, the physical capital market and the labor market, ie equations (B.18) to (B.22).

6. The sequence of endogenous state variables \( \{ N^h_{t+1}, N^b_{t+1}, B_{t+1} \}_{t=0,1,...} \) satisfies the respective laws of motion, ie equations (B.4), (B.5) and (B.14).
C Solution method

The model is solved using global solution methods. In particular, the method used is policy function iteration (Coleman, 1990), also known as time iteration (Judd, 1998). Functions are approximated using piecewise linear interpolation, as advocated in Richter, Throckmorton, and Walker (2014). A sketch of the numerical solution procedure is as follows:

1. Discretize the state variables by creating an evenly space grid, covering the relevant range of values each of them can take. Exogenous shocks are discretized using the method described in Rouwenhorst (1995).

2. Select the set of policy functions. In this case, the variables chosen are $A^h(S)$, $B^b(S)$, $R^D(S)$, $R^B(S)$, and $v(S)$.

3. Specify an initial guess for the policy functions at each point $i$ of the state space (note that the size of the state space equals the product of all the state variable grid sizes) and use them as candidate policy functions. In particular, the initial guess is chosen to be equal to the deterministic steady state value of the selected variables.

4. For each point $i$ of the state space, plug the candidate policy functions into the equilibrium equations and calculate the value of the endogenous state variables at $t + 1$.

5. Using the value of the endogenous state variables at $t + 1$, use linear interpolation to obtain the value of the policy variables at $t + 1$ for each possible realization of the exogenous state variables.

6. Using the value of the endogenous state variables and the policy variables at $t+1$, obtain the value at $t + 1$ of the remaining variables necessary to calculate time $t$ expectations, for each possible realization of the aggregate shocks.

7. Use a numerical root-finder to solve for the zeros of the residual equations, subject to each of the remaining equilibrium conditions. Numerical integration is needed at this step to compute expectations in the equilibrium equations. The result is a set of
policy values in each point \( i \) of the state space that satisfies the equilibrium system of equations up to a specified tolerance level, which characterizes the updated policy function for the next step.

8. If the distance between the candidate policy function and the updated policy values obtained in the previous step is less than the convergence criterion for all \( i \), then the policies have converged to their equilibrium values. Otherwise, use the updated policy functions as the new candidate and go back to step 5.
D Accuracy of the numerical solution

It is possible to assess the accuracy of the numerical solution by computing the residual errors of the equilibrium equations after simulating the model for a given sequence of the aggregate shocks using the approximated policy functions obtained by the numerical procedure described above, as proposed by Judd (1992). To this end, the model is simulated for 200,000 periods. Following standard practice, the decimal log of the absolute value of these residual errors is reported here. Figure D.1 reports the density of these errors.

Figure D.1: Equilibrium equations’ residual errors

Notes: Histograms of decimal log of absolute value of residual errors.
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